REVIEW OF THE PRINCIPLES OF LIFE-OFFICE VALUATIONS

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Our most convinced answers are only questions,....
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THE original purpose of this paper—undertaken at request but not unwillingly—was to review the principles and practice of life-office valuations in the light of modern conditions. It was difficult, however, to deal satisfactorily with the principles of valuation *in vacuo* without reference to more fundamental principles. As a consequence the paper has become more ambitious in its scope than originally intended—and has threatened to run away with itself. The reader will perhaps be less disappointed if he is warned in advance that he is to be taken on a ramble through the actuarial countryside and that any interest lies in the journey rather than the destination.

In the enlargement of its scope the paper has become a more intimate expression of a personal point of view, and more controversial. For this reason it is desirable to emphasize that the views expressed must not be taken as necessarily representing the views of my colleagues.

For the same reason I have felt it best to make occasional use of the first person singular. It would be less than fair to give an air of finality to views which are personal and which indeed are still developing.

1. NATURE OF OUR ESTIMATES

The foundations of actuarial practice were laid before the 1914 war. Since then there has been a considerable change in general outlook, the magnitude of which time may prove to have been greater than we now realize. Before 1914 there was the promise of uninterrupted stability in economics and social affairs, and there was apparently much justification for faith in the unlimited progress of science and the ultimate conquest of all our problems. To-day, what then seemed most certain has in many ways proved to be most uncertain, and in science the rigid and somewhat arid era of physical determinism has given way to a more flexible philosophy with probability and statistics as the prominent factors.

In our own sphere this growth in uncertainty has been emphasized by the different financial consequences of two world wars, the first of which was accompanied by a rise in the long-term gilt-edged rates of interest from 3% to 6% and the second by a fall from $3\frac{1}{2}\%$ to less than $2\frac{1}{2}\%$. In our early literature it was mortality only that demanded treatment by the methods of probability. Now, mortality is perhaps the least of the actuary's uncertainties; interest, taxation and expense, though not susceptible to formal treatment by the methods of probability, are nevertheless factors about which probability must decide the shape of our thinking. We are less concerned about the technique of valuing at $2\frac{1}{2}\%$ than at the significance and the consequences of the $2\frac{1}{2}\%$ itself.

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To begin at the beginning, therefore, what is the nature of our thoughts if we assume—and we have to make some assumption—that the gross rate of interest earned on our funds in 1970 will be 4%? We shall not lose dignity by admitting that we do not know what the rate will be, and we should be unwise perhaps even to use the word forecast.

Behind the assumption is a vague feeling of a continuous series of probability distributions which would express our expectation of the rate of interest at successive points of future time. In the immediate future the distribution would be compact—the margin of uncertainty being small. The uncertainty grows rapidly, however, so that for a point of time, say, five years hence the distribution is more widely dispersed. But the process of widening slows down. Our states of mind concerning the rate of interest ten and thirty years hence are not very different.

Thus we may say that there is an expanding funnel of doubt. The contours of the funnel vary with each one of us, for the concept is personal. Nevertheless, they must inevitably have much in common, since they all start from a common point now.

If we could give numerical values to our uncertainties we could construct a probability dispersion table. For example, we might say that the probability that in 1960 the rate of interest will lie between $3\frac{3}{4}\%$ and $4\frac{1}{4}\%$ is ·15. We cannot, of course, do this in practice. Yet the expanding funnel of doubt is in the background of our thoughts not only in regard to interest but also in regard to the other factors: mortality, expenses, taxation.

2. PREMIUM CALCULATIONS

Whatever may be the convenient working formula finally adopted for the practical calculation of life-office premiums, the original structure may be analysed as follows:

(*a*) the premium for the basic contract, using the best estimate we can make of the future rates of mortality, interest, expense and taxation, without any margins whatsoever and designed to produce neither profit nor loss;
(*b*) a loading for bonus in the form dictated by the requirements of equity and general policy;
(*c*) a loading for contingencies and for variations from the rates involved in (*a*);
(*d*) a loading for options.

In my opinion it does not lead to clarity of thought to make arbitrary exchanges between these groups. It may be convenient for calculation to adopt a simplified working formula, but such a proceeding should never obscure the proper analysis of the rate-structure.

It is not the purpose here to expand on the question of premiums other than is necessary to set the stage for the discussion on valuation. A few brief comments must therefore suffice.

Gross interest. The meaning behind the words 'rate of interest' is discussed later. The only question at this stage is the risk content in the interest on high-yielding securities, e.g. equities. This risk content is a form of insurance premium, and a strong case can be made that it should be deducted from interest and reserved to meet the exigencies for which it was obtained. This comment is made without prejudice to the particular system of account-

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ing to be adopted. It is assumed in all that follows that the risk-premium has been so deducted from interest and put to reserve.

Taxation. The future rate is a matter of opinion, but so long as the basis remains as at present it seems essential to deduct full tax separately from interest and expenses—at least in the Ordinary branch. To deduct from interest alone the net rate of tax payable on interest less expenses is not only unrealistic and inequitable but dangerous, since, in these days of inflated new business, expenses are high and the net rate of tax is low. The possibility that the basis will be altered must be relegated to the list of miscellaneous contingencies.

Bonus loadings. The rate of reversionary bonus to be allowed for is not dealt with in this paper, although the later sections discuss much that is relevant to the decision.

Options. The concealed danger of options, both in assets and liabilities, is a recurring note in this paper. In theory, options should be charged for, and the option premiums received should be accumulated in a fund to provide the cost of meeting the options when they are selected.

Contingencies. In principle, contingencies are of two main forms, although the separation cannot always be made in practice: chance variations about the mean rate, and variations in the mean rate itself.

If the probability dispersion table were taken as a model a strict theory could be constructed, but in practice our thoughts must be largely intuitive, since the table is itself no more than a vague background. Furthermore, there are many other factors to be considered. For example, the matching of investments and liabilities can reduce the need for contingency loadings. The main features are, however, that contingency loadings should be related to the risks of serious loss, bearing in mind the size of the office's reserves, and that on receipt they should be reserved against the contingencies for which they were charged. The normal process would be a steady accumulation of contingency loadings into contingency funds and a withdrawal therefrom to meet emergencies as they arise.

A subsidiary question is that the contingency loadings for individual policies should be related to the degree of doubt expressed in the dispersion schedule. It would be academic to pursue this in detail, but the general principle emerges that with the 'expanding funnel of doubt' there is more risk attached to long-term than to short-term policies. I am inclined to think that in general we tend to undercharge long-term non-profit contracts and overcharge short-term policies.

The question whether with-profit policies require contingency loadings or contingency funds is embedded in the principles and methods of valuation and bonus distribution. Contingency funds may be required for with-profit business as for non-profit, though perhaps not equally; but it is rather academic to discuss whether it is better, for example, to load for 35*s.*% bonus or to load for 30*s.*% bonus with a 3*s.*% addition to the premiums for contingencies.

3. MATCHING OF INVESTMENTS—IMMUNIZATION

The word 'matching' implies the distribution of assets to make them, as far as possible, equally as vulnerable as the liabilities to those influences which affect both. In its widest sense this principle includes such important aspects as the matching of assets and liabilities in currencies.

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In the narrower sense to which the remainder of this section applies, 'matching' implies the distribution of the term of the assets in relation to the term of the liabilities in such a way as to reduce the possibility of loss arising from a change in interest rates. This aspect of life assurance is of the greatest importance. It is one of the three main factors within our control which can endanger the solvency of an office (the other two being the covering of war risks and the granting of guaranteed option values, particularly surrender and settlement options). 'Matching' fundamentally affects and should be affected by the bonus policy of the office. The day-to-day investment of a life fund brings many practical problems, but it is important that in the exigencies of the work the general goal should not be overlooked.

The subject has recently been profitably discussed by Suttie, Whyte, and Coe and Ogborn (in the text-book shortly to be published). It is fair to say, however, that there is a considerable variety of view within the profession, and that there is nowhere in our literature a precise and accepted statement of principle. I find it desirable therefore to submit a theoretical basis as a springboard for discussion.

The word 'matching' has such a wide and general connotation that it is necessary to adopt a new label with a more precise significance. For this purpose I use the word 'immunization' to signify the investment of the assets in such a way that the existing business is immune to a general change in the rate of interest. The definition is not exact, but it should not mislead. On the basis of this definition immunization is to be regarded as a particular form of matching.

Adopting certain convenient simplifications of the practical problem, it will be assumed that, at a given moment of time, securities can be obtained to yield a uniform rate of interest whatever the term, and that all the funds are invested in fixed-interest securities which are either irredeemable or redeemable at a fixed date. For the moment it will be convenient to treat bonuses as guaranteed and included in the benefits.

Symbolically, the problem presents no difficulties.

Let L_t be the expected net outgo of the existing business in calendar year t, viz. claims and expenses less premiums (described hereafter as liability-outgo).* L_t can be positive or negative. For a growing company it will usually be negative in the near future and positive later.

Let A_t be the expected proceeds from the existing assets in year t, viz. interest plus maturing investments (described hereafter as asset-proceeds). It will be seen later that these definitions are crucial to a simple solution of the problem—and particularly the inclusion of interest in asset-proceeds. It is sounder to treat interest as part of the asset-proceeds rather than as a deduction from liability-outgo. In this way the liability-outgo is a fixed element in the investigation independent of investment policy. To deduct interest from the liability-outgo is to confuse the dependent and independent variables. Indeed, these definitions point the way directly to the main part of the solution; in the broadest sense, it is apparent without mathematical proof that if the liability-outgo and asset-proceeds are to be equally sensitive to changes in the rate of interest they must have roughly the same mean

* (1) The incidence of tax is assumed to be appropriately allowed for, e.g. tax is deducted separately from expenses and interest in the U.K. (2) Surrenders are ignored, the assumption being that the values granted are kept within the amounts available.

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terms. This intuitive solution is not quite sufficient, as will be seen later, but is mentioned here to give the reader a common-sense foundation to his appreciation of the mathematical passages which follow.

Let V_L be the present value of the liability-outgo at the ruling rate of interest (force δ), so that $V_L = \Sigma v^t L_t$. Let V_A be the present value of the asset-proceeds at the same rate of interest so that $V_A = \Sigma v^t A_t$. Let it be further assumed that at the present moment $V_A = V_L$, any excess assets being 'free' funds to be separately invested.

Now suppose that the force of interest changes from δ to $(\delta + \epsilon)$ with a consequent change of V_A and V_L to V'_A and V'_L. Then the position after the change of interest is given by Taylor's theorem:

$$V'_A - V'_L = (V_A - V_L) + \epsilon \frac{d(V_A - V_L)}{d\delta} + \frac{\epsilon^2}{2!} \frac{d^2(V_A - V_L)}{d\delta^2} + \ldots.$$

The first term in the expansion vanishes since $V_A = V_L$. It is clear that if there is to be no profit or loss whatever from the change in the force of interest then all the successive derivatives must vanish. In practice the first derivative is the most important for small changes of the rate of interest, and I shall therefore define a fund as immunized if the assets are so invested that $\frac{d(V_A - V_L)}{d\delta}$ is zero.

If the second derivative is positive, then, since the coefficient $\epsilon^2/2!$ is positive whether ϵ is positive or negative, any change in the force of interest will result in a profit to the fund so long as the change is not so large that the higher terms in the expansion begin to take effect. It is desirable, therefore, although the illustrations given later show that the point is not of great importance, that $\frac{d^2(V_A - V_L)}{d\delta^2}$ should be positive.*

A satisfactory immunization policy can, therefore, be expressed symbolically in the two equations

$$\frac{d(V_A - V_L)}{d\delta} = 0, \tag{1}$$

$$\frac{d^2(V_A - V_L)}{d\delta^2} > 0. \tag{2}$$

These expressions can be given verbal interpretation by expanding as follows:

$$V_A = \Sigma v^t A_t,$$

$$\frac{dV_A}{d\delta} = -\Sigma t v^t A_t,$$

$$\frac{d^2V_A}{d\delta^2} = \Sigma t^2 v^t A_t,$$

and similarly for V_L.

Thus equation (1) can be expressed as

$$\Sigma t v^t A_t = \Sigma t v^t L_t, \tag{3}$$

or, verbally,

> the mean term of the value of the asset-proceeds must equal the mean term of the value of the liability-outgo.

* By analogy with equilibrium in statics we could describe a fund as immunized if the first derivative is zero, and the immunization as stable or unstable according as the second derivative is positive or negative.

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Equation (2) can be expressed as

$$\Sigma t^2 v^t A_t > \Sigma t^2 v^t L_t, \tag{4}$$

or, in broad terms,

> the spread of the value of the asset-proceeds about the mean term should be greater than the spread of the value of the liability-outgo.

The simplicity of these formulae is due to the inclusion of interest in the asset-proceeds, but a further step has to be taken before the principles can be translated into a form which produces asset-maturity dates.

To illustrate the argument, consider the simple case where the liability-outgo consists of a single payment of one unit in 10 years' time. If the rate of interest is $2\frac{1}{2}\%$ net, the present value of the liability, $V_L = v^{10} = \cdot 78120$. This is also the present value of the asset we have to invest. There are an infinite number of ways of investing this asset to satisfy equation (3). If we could find an asset with no running yield (e.g. a savings certificate) maturing in 10 years time then we could immunize the liability by such an asset without further complication. But if, as is generally the case, we have to employ interest-bearing assets the term of the asset must be longer than 10 years, so that the mean term of the maturing investment plus the interest income equals 10 years. If we take the simplest case of investing in a single security yielding $2\frac{1}{2}\%$ net we find that the term of that asset must be 11·485 years. The mean term of the value of the total asset-proceeds (viz. interest of ·01953 for 11·485 years plus the redeemed asset of ·78120) is 10 years—the same as the mean term of the value of the liability-outgo. This illustration establishes an important theoretical and practical point that the term of the asset-maturity dates will be appreciably longer than the term of the value of the asset-proceeds and of the liability-outgo—but see (*c*) below. As a rough guide, for a given investment maturing in N years, if the interest-income gives a running yield at the same rate as the rate, i, at which the calculations are being made, then the mean term of the value of the asset-proceeds from that investment is $\bar{a}_{\overline{N}|}$. Put the other way round, if we have a particular liability due in T years' time, it can be immunized by an investment maturing in N years time where $\bar{a}_{\overline{N}|} = T$. For example, if $i = \cdot 025$ and $T = 20$ then N is about 28 years. This example illustrates the lengthening of the asset-maturity compared with the asset-proceeds term. Another feature is disclosed by this illustration: There is a maximum value for T, viz. $1/\delta$, at which $\bar{a}_{\overline{N}|}$ is a perpetuity and N is infinity. If T is greater than this then there is no real solution for N. It is to be noted that in this paragraph the assets have been assumed to be invested in securities with the same running yield as the rate at which the calculations are made. If there were no running yield (e.g. savings certificates) the asset-maturity date would coincide with the mean term of the value of the asset-proceeds because the mean term of the proceeds of an asset with no running yield is the same as the term to maturity.

Examples of immunization and comments thereon are given in Appendix 1.

In regard to the basic equations (3) and (4), a few general observations can be made.

(*a*) There are usually an infinite number of solutions to the two equations, although, as has been seen, there may be no real solution for the asset-maturity dates.

(*b*) The interest-income included in the asset-proceeds represents no particular rate of interest. It is merely the income from the investments included in the fund, whether they were bought above or below par, or even bought in the distant past when quite different yields were obtainable.

(*c*) The equations define the position at a moment of time. Their solutions change continuously. For a fixed block of business followed through its history the difference between the term of asset-maturities and that of liability-outgo diminishes with the passage of time until, when the final liability payment falls due, it has become zero.

(*d*) The solution of the equations is dependent upon the current rate of interest.

(*e*) Changes in the constitution of the business inevitably affect the solution of the equations. For this reason, and also for the reasons given in (*c*) and (*d*), continual changes will be required in the investment portfolio. This does not invalidate the theoretical answer, since the necessary changes can be made without impairing the basic equation $V_A = V_L$.

(*f*) For a fund which without further new business would decline there is a particular theoretical solution to the equations when the asset-proceeds are arranged to coincide absolutely with the liability-outgo. This solution makes all the terms in the Taylor expansion zero and is therefore absolutely immune to all changes in interest however violent. This special case of immunization can be described as 'absolute' matching.

A sceptical reader may ask how it is possible to immunize existing business when, even if no new business is written, the funds will continue to grow for some years and require the investment of that growth at unknown rates of interest. The verbal answer to this question is that, if the rate of interest falls, there will be a shortfall in the yield on the future investments which have to be made, but there will be an exactly balancing excess in the appreciation of existing investments which have been invested for longer terms than the liabilities they have to meet. And similarly *mutatis mutandis* for a rise in the rate of interest.

This section may be fittingly concluded with a summary. The essence of the immunization theory is contained in two definitions, two rules and a rider. The definitions are 'liability-outgo' and 'asset-proceeds' and need not be repeated. (It is assumed that the present values of the two are equal.)

Rule 1. The mean term of the value of the asset-proceeds must equal the mean term of the value of the liability-outgo.

Rule 2. The spread about the mean of the value of the asset-proceeds should be greater than the spread of the value of the liability-outgo.

Rider. The mean term of the asset-maturity dates is considerably greater than that of the value of the asset-proceeds.

To these rules there should be added a clear warning that the whole examination is theoretical. Not only are there many and serious difficulties in giving practical effect to the theory (amounting in many circumstances to impossibility), but the extent to which it would be wise in practice to adopt the theory is a matter for consideration. It has to be remembered that apart from the minor second-derivative profits the immunization is against profit as well as loss.

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4. CONSEQUENCES OF IMMUNIZATION

Let us consider a fund immunized at a rate of interest of $2\frac{1}{2}\%$ and assume that for the time being the rate of interest remains unchanged. The term of the maturities of the investments will usually be very long for a young fund. As the fund ages, this term shortens. If the fund becomes younger by heavy influx of business the term lengthens again. But so long as the fund does not become so very young that the term reaches infinity, the whole of the business, including any new business, can theoretically be immediately immunized at the $2\frac{1}{2}\%$ rate of interest. For a fund kept stationary by the influx of new business the theory entails a steady process of selling shorts and buying longs in order to maintain the same mean term.

Let us consider what happens when there is a change in the rate of interest. Any existing non-profit business has fixed benefits and premiums and is assumed to have been immunized at the $2\frac{1}{2}\%$ rate of interest. For with-profit business we can regard the bonus as guaranteed for this purpose, and it will have been immunized at the rate of bonus supported by a $2\frac{1}{2}\%$ rate of interest. If the interest changes to $3\frac{1}{2}\%$ the mean term of the value of the liability-outgo will change slightly, and a small rearrangement of the portfolio will be required which will give rise to a small 'second-derivative' profit or loss, although substantially the existing business will be unaffected. The benefits and premiums and the old guaranteed rate of bonus remain unchanged. When this rearrangement has been made the whole fund is immunized on a $3\frac{1}{2}\%$ basis, and the fact that it was once on a $2\frac{1}{2}\%$ basis can be forgotten except for the one fact that it is still the rate of bonus on the basis of $2\frac{1}{2}\%$ interest which has been immunized.

Now let us consider theoretically what happens when new business enters immediately after the rate of interest has changed. Considering non-profit business first, it is easier to see what happens if it is assumed that the rates of premium are changed from a $2\frac{1}{2}$ to $3\frac{1}{2}\%$ level, so that for this new business at the new rates of premium and the new rate of interest the premiums exactly match the benefits. If the investments for the whole fund continue to be made so as to fit the immunization equation, this new business will be completely immunized at a $3\frac{1}{2}\%$ level. For with-profit business we can reduce the premiums so as to produce the same bonus as the existing business. If we do this, the whole business, old and new, is immunized at the old rate of bonus. Or we can keep the old rate of premium which will now support a higher rate of bonus (say 45*s*. compared with the previous 30*s*.) and the new business will then be immunized at a 45*s*. rate of bonus.

The mathematical consequences of following this immunization procedure would be far-reaching. For well-established funds the theory brings these conclusions:

(*a*) That even though assurance contracts run for long periods into the future they could generally be immunized at the rate of interest ruling *at the date of issue* by immunizing the whole fund, including existing business and new business. (It is to be remembered, however, that it is the presence of the existing business which makes the immunization of new business possible.)

(*b*) That for non-profit business it is mathematically appropriate to change the rates of premium immediately upon a change in the general level of interest.

(*c*) That if a uniform rate of bonus is to be maintained it is also theoretically

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reasonable to change the with-profit premium rates immediately upon a change in the rate of interest. The theoretical alternative would be, while maintaining the same rates of premium, to differentiate the bonus according to the date of issue of the policies.

These mathematical results may be surprising and their appropriateness in practice may be questioned. As has been explained, the purpose of this section on immunization is not to advocate a system of investment, and it is perhaps desirable having proceeded so far to mention some of the practical complications:

(*a*) There is difficulty in relating varied yields on assets to some particular rate of interest for valuing liabilities. Yields are not uniform for all terms of assets, and the differentials are not stable in time. Nor are the differentials between classes of assets stable.

(*b*) The theory is considerably disturbed by the wide range of redemption dates contained in many Government securities. Options, whether in the assets or the liabilities, can be serious danger points.

(*c*) The theory would be difficult to interpret in practice because of the existence of such assets as equity shares, properties, mortgages on an open basis, and so on. Either the income, or the term, or both, of many assets are indeterminate.

(*d*) Offices would be reluctant, and properly so, to invest heavily in long-term securities in a period of very low interest rates.

(*e*) Investment policy must be flexible to take advantage of special opportunities.

(*f*) Offices have considerable surplus funds, which can be regarded as falling outside the immunization theory.

It may well be concluded that the immunization theory should not dictate investment policies, although it is enlightening about the consequences and points to a norm.

In addition to the difficulties of implementing the theory there are problems of the public presentation of results. These problems are discussed in the later sections on the valuation of assets and liabilities. There are some aspects, however, which can best be dealt with here.

Immunization implies that a fall in the value of assets would be matched by a fall in the value of liabilities, but in published results this will be realized only if the system of valuing the liabilities follows the logic of the theory, that is to say, if it is a gross premium valuation of the anticipated actual income and outgo. If some other system of valuation is used then the correspondence between the fall in assets and liabilities will not necessarily be apparent. For example, net premium valuations are far less responsive to changes in interest rates than gross premium valuations. It is true that by investing relatively short the sensitiveness of assets to a change in the rate of interest could be made to correspond to the comparative insensitiveness of liabilities valued on a net premium basis. But this is not genuine immunization. A particular valuation basis may be desirable for many reasons but it must be a servant of realities for it cannot be their master.

There is a point to notice, however, in connexion with practical valuations. If the rate of interest rises, a valuation of liabilities at this higher rate may produce negative values. The immunization theory implies that these negative values are real, whereas policy-holders may exercise their option to lapse.

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This is a particular example of the general option problem. There is no theoretical solution, but it might be reasonable for the younger funds, where negative values may be substantial, to invest somewhat short of the theoretical asset-maturity dates.

5. VALUATION PRELIMINARIES

The following comments, amounting almost to definitions, will simplify later sections.

Gross rate of interest

The gross rate of interest apparent from a life assurance revenue account may, of course, be misleading. The interest shown in the account may include interest on capital and contingency funds which are themselves not included in the 'life assurance fund'. A more realistic rate would be obtained by dividing the total interest by the total assets shown in the balance sheet. But this is still misleading, since the balance-sheet value of the assets may differ from the market value—if, indeed, market value is itself a final denominator. Further to confuse the issue, as the value of the assets changes under changing conditions so should the amount of interest. For example, the value of a debenture yielding 4%, redeemable in 10 years and standing at par, may rise to 108 upon a general fall in interest rates to 3% and still give an apparent yield of £3. 14*s.*; but this yield is misleading, since the interest receipts include part of the extra capital which will be written off over the term to maturity. Finally, there is normally a considerable range between the interest yields on long-term and short-term investments and between the various categories of security. In these confused circumstances, what do we mean by 'the' rate of interest?

It is a tempering thought that, at the very foundation of the subject, the main factor should be so elusive. There is, I believe, no easy remedy. When the actuary values the liabilities at a gross rate of interest of, say, 4%, his meaning is clear enough, but it will need internal investigation to verify that this rate has a reasonable prospect of being maintained.

The principle that for redeemable assets the amount of interest should change as the value of assets changes is fundamental and forms an undercurrent to the whole valuation problem. Our thoughts on the subject would be more efficient if we had a more precise set of verbal tools. We should adopt some such phrase as 'income from investments' to cover the current income which appears in our revenue accounts as 'Interest, Dividends and Rents'. The word 'Interest' is dangerous, and a broader title would make it clearer that this income has no specific relationship to any valid 'rate of interest'. In the example given above the only true figure at that moment of time for a rate of accumulation is 3%, and there are obvious dangers in allowing thoughts of 4% (if the book value is kept at 100) or of £3. 14*s.* 0*d.*% (if the book value is written up to 108) to enter our calculations. We must always distinguish between g and i.

Income tax

In order to make proper allowance for tax on 'interest less expenses' I believe that the full rate of taxation (now 7*s.* 6*d.* in the £) should be deducted from both items and that this should be done not only in premium calculations

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but also in valuation. This has been disputed, but it is doubtful if a certificate of solvency should be given to a fund which would not stand a test on this basis. To do less is to rely on a subsidy from future new business. (For industrial business the practical basis of taxation is often 'proprietors' profits', and the existence of the alternatives raises a complex technical issue which cannot be discussed here.)

Estate

A life office may, and usually will, have some margins or reserves in three places:

(*a*) within the valuation of liabilities, either explicit or implied in a stringent valuation basis,
(*b*) in asset-margins, and
(*c*) in central reserve or contingency funds or capital.

It is convenient to give a title to all these reserves combined, and the word 'estate' has a useful connotation.

It is illuminating to examine the distributed surplus for a valuation period under the following three headings—without too close examination of the words at this stage.

		£
	(i) True surplus earned by existing business	.....
plus	(ii) Interest contribution from the existing estate	.....
less	(iii) Contribution from surplus to the estate	.....
	Total (= distributed surplus)	

These three items alone form an interesting commentary on the inner history of a life assurance fund and go a long way to rectify any confusion arising from conventional presentations.

New business expenses in valuation

It has become conventional in this country to allow, directly or indirectly, for future expenses at the over-all level experienced by a continuing fund and to ignore the fact that, new business expenses having already been incurred, future expenses on the existing business will be renewal expenses only. The consequence is usually (depending upon the precise method of valuation) that new business causes a heavy strain, and that the resulting surplus is considerably distorted.

There is no question but that the distortion may obscure the truth of events, even for the actuary himself, unless correction is made. For internal purposes I think it is essential therefore to use a modified valuation basis which eliminates this distortion as far as possible.

For a net premium valuation the phrase 'unmodified net premium valuation' will be used to denote a valuation employing the unloaded net premium, P_x, and the phrase 'modified net premium valuation' to denote a valuation employing net premiums adjusted for new business expenses ($P'_x = P_x + K/\ddot{a}_x$, where K represents the additional new business expenses per unit sum assured, net of tax, which it is felt can be appropriately spread over the future).

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It is well known that (where premiums are payable throughout the full duration)

$$V'_x = V_x - K(S - V_x),$$

and, in the usual way, $K(S - V_x)$ is referred to as the Zillmer deduction, and V_x as modified or Zillmerized reserves, etc.

Although it is well appreciated that new business strain can arise under an unmodified net premium valuation, it is not equally appreciated that it can also arise under a gross premium valuation. Discussion has taken place in the past as to whether under a gross premium valuation future expenses should be allowed for at the over-all level or the renewal level. The best answer is probably neither, because if the fund were closed to new business the rate of renewal expense would rise. Subject to this qualification, however, the rate of expense employed in a realistic valuation should approximate to the renewal level rather than the over-all level.

The phrases 'unmodified' and 'modified' will be applied to gross premium valuations with the same significance as for net premium valuations. Where no specific reference is made it is to be understood that modified valuations are being considered.

There may be the soundest reasons for holding additional reserves and the use of the over-all level of future expense may be an excellent way of holding such reserves, but for internal purposes, in trying to arrive at the true trend of events, renewal expenses only must be debited against business on the books. The acceptance of new business strain may be a pleasant luxury or even a near-necessity, but it is better to regard it as an adjustment made after the surplus has been examined and not before.

Changes of basis

It is a commonplace that the basis of valuation does not directly affect the present value of the surplus which will ultimately accrue but merely the incidence of the emergence of surplus. In Appendix II this is illustrated by the separate histories of a given block of business controlled by different valuation bases. Theoretically, a strong valuation basis dams up the surplus and provides a deeper reservoir.

It is important, however, to consider what happens when the valuation basis is changed—for example, when it is weakened by an increase in the rate of interest employed. The theoretical result is a large release of surplus in the year in question associated with smaller surpluses in all future years. In practice, however, the outcome depends on the use which is made of the large release in the year in question; if, in fact, it is reserved and not distributed it will continue to earn interest and to contribute to surplus. The ultimate effect will depend on the future history of the office. If the business declines, the office may ultimately prove to be stronger by reason of having weakened its valuation basis and having put the release to reserve than if it had maintained a stronger valuation basis and distributed the resulting higher surplus as bonus.

This train of thought has to be borne in mind throughout the rest of the paper. Under changing conditions there is a continual interchange between current liabilities and the estate, just as there is between capital and interest in the revenue account.

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6. VALUATION PRINCIPLES

The preceding sections of the paper have been in the nature of a preliminary to a consideration of the problems of valuation proper. They have dealt primarily with the underlying realities, whereas valuation is to a considerable extent a problem of presentation.

A valuation has two main purposes, and the fundamental difficulty is that these two purposes are in conflict. The first and primary purpose is to ensure that the office is solvent. The second is to allow the surplus to emerge in an equitable way suited to the bonus system. The solvency criterion leads to a changing valuation basis, influenced solely by prospective considerations and probably uniform for all policies. On the other hand, the pursuit of equity of emergence of surplus tends to lead to stable valuation bases, influenced mainly by retrospective considerations and possibly differentiated according to the terms at issue. We are thus faced at the beginning with that most fruitful source of controversy, the attempt to reply to two different questions in a single answer.

A third main purpose of valuation should perhaps be added: the presentation of results in a proper manner for public scrutiny. This duty may be troublesome when so much of the problem is complex and technical; but it is probably, in the long run, in the best interests of the offices.

There is a further fundamental conflict: the valuation of assets and the valuation of liabilities have a different rationale. Assets and liabilities are different in nature, the value of the former being more speculative, but this difference is not sufficient to justify the odd position into which history has led us. The customary test applied to the valuation of assets is a test appropriate to a solvency valuation, namely, 'Are the assets of the value stated?' On the other hand, the valuation of liabilities has, in the course of time, acquired most of the attributes of an equity valuation. It is an interesting train of thought to consider what the valuation process would be if we adopted a similar basis for both assets and liabilities.

Fig. 1 shows the same problem in a different light. This diagram illustrates for a hypothetical office the value of assets and liabilities at different rates of interest. The heavy lines A and L show the position if there were no options. The dotted lines show the effect of various options.

A is typical of the effect on the value of assets if the fixed-interest securities have a wide range of redemption dates at the option of the borrower.

L' shows the effect of options to take cash, e.g. guaranteed cash surrender values, or options with a smaller interest content than the basic contract.

L'' shows the effect of options with a larger interest content than the basic contract, e.g. annuity options at maturity.

The office shown in Fig. 1 is, as the result of holding properly matched investments, basically sound because the assets exceed the liabilities at all rates of interest, but the dotted lines show how the effect of options can whittle away the surplus, especially in the extreme conditions of high and low interest rates.

Figs. 2 and 3 illustrate the effect, in two offices with the same liabilities, first of investing short, and second of investing long. Both these diagrams show a similar substantial surplus if the gross rate of interest is 4% but show that insolvency would result if the rate of interest falls low enough for the first example, or rises high enough for the second.

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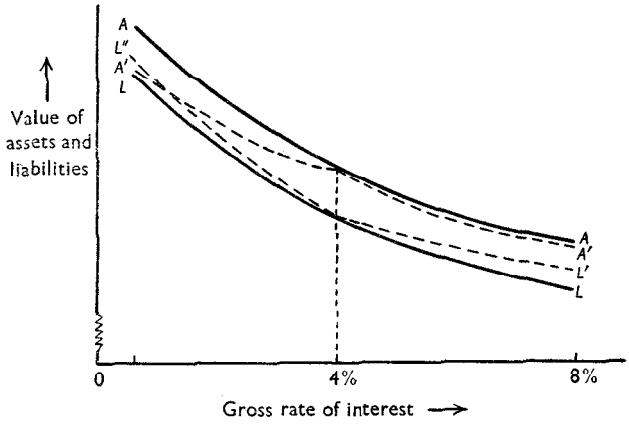

Fig. 1

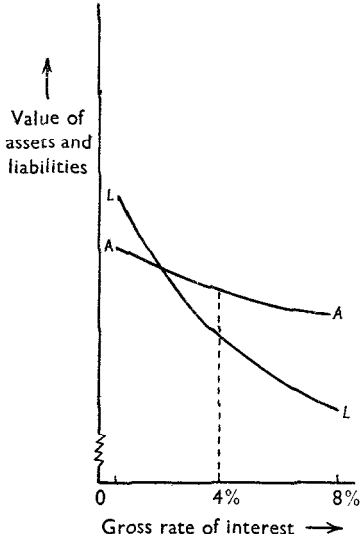

Fig. 2

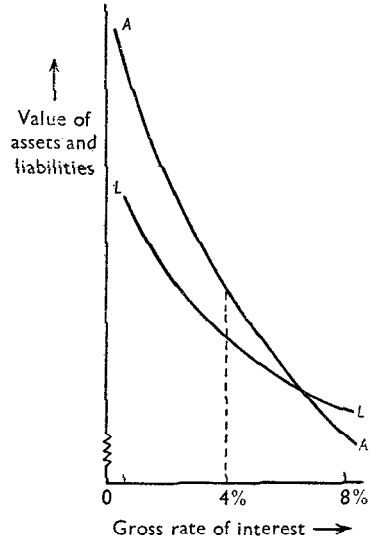

Fig. 3

Fig. 1. Illustration of the variations in the values of assets (A) and liabilities (L) with the rate of interest when the fund is immunized, and of the effect of options.

Fig. 2. Illustration of the variations in the values of assets (A) and liabilities (L) with the rate of interest when the fund is invested short.

Fig. 3. Illustration of the variations in the values of assets (A) and liabilities (L) with the rate of interest when the fund is invested long.

These two offices are only conditionally solvent. The office in Fig. 1 could almost be described as absolutely solvent.

It should be explained that because of the difficulties of locating the rate of interest these graphs can hardly be constructed in practice. We cannot precisely fix the A-curve in relation to the L-curve. We can, however, obtain a fair idea of the slope of the two curves, and this is sufficient to give us considerable insight into the anatomy of the fund.

These diagrams, which are fundamental, contain the essence of the valuation problem.

7. WHAT IS SURPLUS?

This and the next three sections deal with the interlocking of valuation practice and surplus. They do not deal with the question of how much of the surplus should be distributed in bonus or how the amount so distributed should be apportioned among various policy-holders.

The question 'What is surplus?' is even more elusive than the question 'What is the rate of interest?' In my opinion, it is essential to keep a clear distinction between two aspects of surplus which for convenience I shall call 'revenue surplus' and 'capitalized surplus'. An explanation of these terms is best given by an illustration. Let us suppose that the mortality experience, while showing the usual random variations from year to year, discloses a declining trend over a long period. A time may come when we decide that the basis of mortality employed in valuation in the past must be lightened. Although the facts move gradually and certainly, the decision is both sudden and speculative. The change to a lighter mortality basis in valuation will have an immediate effect on surplus in the year in question—let us suppose a release. This release should be described as capitalized surplus. It is due, not to the favourable experience during the inter-valuation period as compared with the valuation basis, but to the decision to change the valuation basis for the future expected experience, thus anticipating surplus which would otherwise have accrued in the future. This distinction between capitalized and revenue surplus has parallels in the distinction between capital and revenue in company accounts and indeed embraces some of the same points, since revenue surplus for a life assurance company can only be ascertained if capital items are excluded from the revenue account as well as from the valuation results.

It would perhaps be tendentious to apply the label 'true' to any particular figure for the surplus, but there is one figure for the surplus which can legitimately claim the title of 'natural', namely, surplus resulting from a valuation on the same bases as were used in the calculation of the premiums and based on revenue accounts from which all capital items have been scrupulously excluded. Surplus calculated by this system can be described as 'natural revenue surplus', and a valuation on the premium basis can be described as on the 'natural valuation basis'.*

* It may be that with-profit premium rates are left unchanged for many years, and that the nominal formula employed becomes obsolete and out of step with current experience. It is desirable, however, to re-express the premium rates actually charged in the light of the actuary's assessment of the future experience at the time of issue. The re-cast formula expresses the plan or intention at the time a policy is issued, and it is a valuation on the basis of this original plan which earns the title of a 'natural valuation'.

The concept of 'natural revenue surplus' is fundamental to the appreciation of the following sections. Some comments may point the issue. There may be—indeed, there often are—good reasons why an office should capitalize future losses even perhaps at a time when their occurrence is still only problematical. It nevertheless remains desirable for clarity of thought that we should distinguish between such capitalized losses and the natural revenue surplus. Here again we note the conflict between a solvency valuation and an equity valuation. 'Natural revenue surplus' corresponds essentially to an equity valuation, whereas our instinct to capitalize future losses is a reflexion of our proper desire also to conduct a solvency valuation. A valuation on the premium basis has no direct connexion with solvency and could in exceptional circumstances leave the office insolvent.

A final illustration may help. Let us consider the position of an office which has not matched its assets to its liabilities and has, for example, invested short. On a change in the rate of interest, the failure to match will result in large profits or losses if any attempt is made to capitalize the situation as it appears at the moment of valuation. These profits or losses may in fact never materialize if the rate of interest reverts to its original level. The surplus under such conditions would fluctuate, and what we think of as profit at one moment may ultimately prove to be a loss. If, however, for that same office the valuation of liabilities is conducted on the natural valuation basis and the valuation of assets is unchanged, the consequences of the failure to immunize affect the surplus as and when they materialize—namely, when the over-short investments having matured and fresh investments having been made, a higher or lower rate of interest than originally anticipated is physically received. This illustration shows that whatever qualifications may need to be attached to the natural revenue surplus as a practical instrument it is at any rate a clear conception, whereas any other estimate of the surplus is not.

8. VALUATION UNDER STABLE CONDITIONS

For with-profit premiums which include a bonus loading, a valuation on the natural or premium basis would normally be described as a bonus reserve valuation. The phrase 'bonus reserve valuation' usually implies a gross premium valuation, but it is of course possible to have net premium bonus reserve valuations. A valuation on the same basis as that employed in the premiums can equally well be described as a gross premium bonus reserve valuation or as a net premium bonus reserve valuation, since the two are identical.

It is well known that the conventional net premium valuation basis adopted by most British offices can be made to approximate closely to a bonus reserve valuation on the premium basis by employing a Zillmer factor and by an artificial reduction in the rate of interest. The table in Appendix II shows the emergence of surplus for a given block of new business on various valuation bases, and it is seen that the emergence of surplus under basis *b* (iii) is very similar to that on the strict premium basis.

If these were the only considerations the bonus reserve method of valuation would appear to have the advantage over the net premium method; the bonus reserve valuation is direct and self-explanatory, whereas the net premium valuation is artificial. Further, a gross premium bonus reserve valuation, not requiring a net premium constant, is simpler.

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There is, however, an aspect of the problem which does not appear in a theoretical examination but greatly affects the practice of valuation. Premium bases are delicate instruments which can and do include refinements to allow for all the many factors influencing the actuary's judgment in fixing his premium scales. They can, for example, allow for a different rate of interest for different terms of policy, for different mortality adjustments at different ages and so on. It is not practicable to carry these refinements into a valuation. Expediency demands that a simple valuation basis be adopted which applies generally to policies issued in the past under quite different premium bases. The difficulty of a bonus reserve valuation is that it immediately capitalizes any difference between the premium basis and the valuation basis; it inevitably introduces a capitalized element into the revenue surplus. If, for example, we consider the simple case of a block of new business, it is not possible to value each policy on its individual premium basis; an aggregate basis has to be used, and this aggregate basis may produce a substantial and immediate new business strain. It is not easy to identify this new business strain, and it can produce a substantial distortion of surplus. The modified net premium valuation has an advantage in this respect, since the emergence of surplus thereunder is not seriously distorted even if the valuation basis does differ slightly from the premium basis.

The great justification for the net premium valuation is that it gives an approximation to the premium basis of valuation and therefore to the natural revenue surplus. As such it is relatively fool-proof and cannot easily be abused. It cannot be used, for example, to capitalize future profits and thereby to bolster a position where expenses are running at an untenable level. When we consider that the life assurance fund of a well-established company contains the history of many years of changing events, there is great virtue in a net premium valuation which can be made to approximate to all the various premium bases which have been employed.

The view is sometimes held that in some way a bonus reserve valuation is scientifically accurate, whereas the net premium valuation is no more than a convenient façade. It is odd that those holding such an opinion do not ask themselves the simple question whether it is in fact right to assume a *uniform* future bonus. The answer is clearly negative; changes in the experience and in the constitution of the business should have a direct influence upon surplus. Any claims to some peculiar virtue in a bonus reserve valuation in my opinion arise from a misunderstanding. The bonus reserve valuation is an instrument, like the net premium valuation, with its own uses and its own limitations.

These remarks are not to be interpreted as an attack on the bonus reserve method of valuation. As will be seen later there are circumstances in which the net premium valuation fails us, and when this happens there is much to be said for a bonus reserve valuation. But in stable conditions my personal preference is for a net premium valuation, and I base this preference not upon expediency but upon the personal insight to be gained into the anatomy of a life assurance fund. Three main documents are required to investigate the position of a life assurance fund:

(*a*) An analysis of surplus on a modified net premium valuation basis uniform throughout the period examined. The picture gains in vividness if each item of surplus is expressed as a rate of bonus.

(*b*) Comments on the trend in each of the items in the analysis. The main

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items of information missing from the analysis are the future trends of the loading surplus and interest income resulting from the past history of premium changes and investment practice. With this knowledge it is comparatively simple to estimate the way the fund will unfold in the future if conditions remain unchanged.

(*c*) Some statement of the position of the office regarding the matching of investments against liabilities.

If the valuation has been conducted on a bonus reserve basis, the assessment of the situation is in some ways more difficult; the capitalized surplus or loss which has inevitably crept in has to be excluded. This may not be easy. The future rate of bonus reserved for in the valuation cannot necessarily be regarded as a forecast of future events. It is highly sensitive among other things to the rate of interest employed in the valuation. Similarly, the rate of bonus depends on an estimate of all the valuation factors, including rates of withdrawal. There is a further danger to which all valuation methods are liable, but perhaps particularly the bonus reserve method: if care is not taken to distinguish the estate from the other funds, a valuation which aims to be realistic may disclose excessively high bonus prospects by implying the release of the whole or a large part of the office's estate throughout the lifetime of the present policy-holders.

Some of these difficulties can be avoided by the following internal analysis of a bonus reserve valuation.

On the basis of the best possible estimate of all valuation factors, including provision for withdrawal and bringing in the full market value of assets, let it be supposed that

(*a*) the rate of bonus supportable by existing business using all the existing assets is, say, 52*s*.%,

(*b*) the rate of bonus supportable by current new business is, say, 35*s*.%.

Let it be further supposed that the amount of the reserves released on existing business if the future rate of bonus were 35*s*. instead of 52*s*., is £*X*. This figure of £*X* can be treated as the estate, so that a fair summary of the position of the office is that the rate of bonus supportable in the future is 35*s*., and that to this there can be added such bonus, not necessarily uniform, as the interest on the estate, less any contributions which it is planned to make to the estate, will provide.

This is a clear enough picture, but it has to be admitted that it is laborious to obtain; that it is precarious, since it is highly sensitive to the valuation factors adopted; and that it brings one, uncertainly, to a result which can be readily seen from a net premium valuation. However, I do not wish to exaggerate the difficulties of a bonus reserve valuation. A competent actuary can interpret the results of either a net premium or a gross premium valuation and use them to estimate future trends. The ease and reliability with which he does so will depend more on the scrupulousness of the analysis of surplus than on the method of valuation employed.

9. ACTIVE OR PASSIVE VALUATION POLICY

External conditions are continually changing. Should effect be given to these changes in the valuation?

A preliminary question is 'Can effect be given to them?' and the answer is 'Only very roughly.' A valuation is a crude instrument which cannot

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reflect in detail the variety and complexity of events. There has been a considerable increase in the rate of interest during 1951. But exactly how much? The longs and shorts, gilt-edged, debentures, equities have each undergone a different experience. If we attempt to adopt an active valuation policy and give effect to the year's events by altering both our liability and asset valuations, how are we to ensure that the alteration we make to the one corresponds with the alteration to the other? An error of $\frac{1}{8}\%$ in the rate of interest in a gross premium valuation will entirely distort the surplus.

It must be concluded that an active valuation policy has to be ruled out as a satisfactory means of estimating or controlling the real emergence of surplus. Surplus can be estimated only by a passive valuation policy which leaves the valuation basis both of assets and liabilities unchanged, however much the answer so produced may need qualification in practice. Yet when we turn to the question of solvency there is no word to be said in favour of a passive policy; it is only an active policy, paying full regard to existing (and estimated future) experience, which has any significance. It is this conflict between the solvency demand for an active policy and the equity demand for a passive policy which is the core of our problem.

In practice, the event which brings the problem most forcibly to our attention is a swing in the general rate of interest associated with a swing in the market value of securities. An attempt is sometimes made to side-step the issue by using notional values for certain securities so that in the event of a sharp depression in market values at some 31 December the assets can in total be stated in the balance sheet at a value higher than their market value—the higher value probably being obtained by including redeemable securities at their present value on a lower rate of interest than that ruling at the time. So long as this lower rate of interest is not lower than the rate employed in the valuation then the argument for notional values can be appreciated and in times of sudden crisis their use may have value. In my opinion, however, the use of notional values as a general practice is dubious, and the public instinct to take account of market values of securities has justification. The indiscriminate use of notional values can conceal real insolvency arising from badly matched assets; if the investments are excessively long-dated, the fall in value of the assets will be greater than the fall in the value of liabilities and the loss may prove to be permanent.

One can imagine an intelligent member of the public speaking to this effect: 'I know that any solvency test can only be conditional. It can only say that the fund is solvent in certain specified conditions. But the best I can do is to ask you to certify solvency at the end of each valuation period in the conditions then ruling. It is true that those conditions may not be permanent, but then they may be. If you continue to certify solvency on the basis of the market values ruling at the time of valuation, I shall be satisfied that your successive certificates of conditional solvency amount to a certificate of absolute solvency. If, however, I grant you a dispensation every time you are embarrassed by depreciation I shall always be in doubt. Further, though you may not like doing what I ask it will be good for you too because it will keep you face-to-face with realities.'

The practice which has gradually developed in this country for overcoming the basic difficulty has been to build up reserves to cushion the impact of events. As indicated earlier, such reserves can be held in three places: in the liabilities, either explicitly or implicitly in a strong valuation basis;

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in the assets, by way of margin between book and market values; and in central reserve or contingency funds. When market values fall the asset-margin takes the first blow, and if this is insufficient the central reserves are used to write down assets or to support a balance-sheet certificate which is qualified to the effect that 'the market value of the assets plus reserve funds is greater than the book value'. In such circumstances the liability-cushion cannot be used except by changing the valuation basis and releasing funds from the liabilities to assist the assets. When the wind blows from the opposite quarter, the rate of interest has dropped and market values are high. In this event it is normally only the liability-margin which is immediately available to enable the actuary to sign his report with a clear conscience. It is to be noted, however, that the actuary's certificate as to liabilities is of a less rigorous nature than the market-value test customarily applied to the balance-sheet value of assets; it is a certificate not of fact but of opinion, and while the actuary has the duty not only to satisfy himself but also to give sufficient evidence to make his certificate acceptable to the public, it is also legitimate for him to bear in mind the existence of the other cushions, and if necessary to draw attention to them.

In the earlier days of our profession it was no doubt often the case that the only cushion the actuary could establish—and that with difficulty—was the liability-cushion. It may be that nowadays we are in danger of going too far the other way. There is a great deal to be said for this three-cushion system but it is achieved at the expense of past generations of policy-holders and can be carried to the point of luxury. In these days of rapid inflation even the most mature companies are feeling the strain of new growth, and we have to ask ourselves how far we should ask the present generation to contribute to the estate.

The three-cushion system is wasteful of resources in that the liability-margin and asset-margin are not easily available to meet a difficulty on the other side of the balance sheet. In this respect the central reserve has a great advantage. It can be used to support a qualified balance-sheet certificate, and I would put it to the profession that it could properly be used to support a qualified certificate by the actuary. For example, the actuary could value the liabilities at $2\frac{1}{2}\%$ and add a rider that the reserve funds would enable him to value at 2%. Of course the reserve funds could not be used simultaneously to support both a qualified balance-sheet certificate and a qualified certificate by the actuary. But that is not our problem. As L. Brown recently put it in cogent terms, 'the rate of interest cannot both rise and fall at the same time'.

If at a particular moment of time an office possesses a liability-margin of $£X$ which is sufficient to withstand a drop of $x\%$ in the rate of interest and an asset-margin of $£Y$ sufficient to withstand a rise of $y\%$ in the rate of interest, then—assuming assets and liabilities to be well matched—it can, by placing the two margins in a central reserve fund of $£(X+Y)$, put itself in the position that it can withstand either a rise or a fall of approximately $(x+y)\%$ in the rate of interest. The central reserve fund has a flexibility which the asset and liability-margins do not possess and is an economy which, in the interests of the present generation of policy-holders, we ought not to overlook.

A major item in the liability-margin is the new business strain locked up by the use of unmodified reserves. The use of unmodified reserves is to be respected, and their adoption as a standard of strength by British offices has

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undoubtedly contributed greatly to sound progress. But the reserves so locked up, being largely inaccessible, have not relieved offices from the desire to build up asset-margins and central reserves. We have to ask ourselves at what pace is it reasonable to augment our margins and where is the most serviceable place to hold them. I venture to suggest that unmodified reserves have been an excellent servant but should not be our master, and that circumstances may arise in which justice requires us to depart from them. Would not the equivalent amount held in a central reserve fund be an even better servant?

The broad picture which emerges from these considerations is that a passive valuation policy is desirable in the interests of equity, and that the maintenance of central reserve funds at a reasonable level is an economical way of holding the additional reserves which may be required to demonstrate solvency in changing conditions.

10. MINOR VARIATIONS IN CONDITIONS

The preceding sections have made a case for a passive valuation policy on the natural valuation basis, and have defended the use of the net premium method in practice on the grounds that it is an approximation thereto. It is, however, only an approximation. The time came, for example, when O^M mortality had to be abandoned in favour of A1924–29. On the theory of the natural valuation basis this time arrived, not immediately when the mortality experience confirmed itself at the lower level (whenever that was), but when the bulk of the business had premiums based on A1924–29. In other words, the philosophy of the net premium basis does not call for hurry in giving effect to changes in experience, whether they be in interest, mortality or expense.

This leisurely process in making valuation changes does, however, ignore the demands for a solvency test. It implies that the valuation basis is always likely to be more or less obsolete. In truth it has to be admitted that it is sometimes not immediately apparent from the published returns of a British life office that it could in fact satisfy a rigid solvency test, because strength in one direction, e.g. more than adequate expense margins, has to be offset against weakness in, say, the interest basis. It has to be quickly added, however, that it seldom needs serious examination to discover that the office is indeed handsomely solvent.

In fact, the offices have reached a typically British compromise. In the main they adopt a passive valuation policy allowing the surplus to emerge in what is probably a reasonable approximation to the natural emergence. They do, however, recognize the need for a solvency test by making such occasional modifications in the valuation basis as will serve to satisfy the public that a solvency test could be passed without difficulty. Moreover, it is clear that in the main the cost of these capital changes is—as I think, properly—met out of the company's 'estate' and not out of immediate revenue surplus.

There is a great deal of experience behind the valuation practice of British offices, and while a superficial theoretical examination of that practice might be critical, a more profound examination would conclude that the current practice in this country is at least as satisfactory as the more elaborate attempts at theoretical accuracy which are adopted (or enforced) in some other countries.

The preceding paragraphs in this section have been written with the thought of a net premium valuation uppermost in mind. The position as regards a gross premium bonus reserve valuation is not so clear. The phrase bonus

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reserve valuation tends to imply an active valuation policy. It is apparent, however, that in practice it is used, if not entirely, at least to a considerable extent as another method of passive valuation. Minor changes are customary, but, generally speaking, there is little attempt to give full recognition to the current and estimated future experience at the moment of valuation. With one notable exception, offices using a bonus reserve valuation do not bring in their assets at market value, which may be regarded as the hall-mark of a truly active policy. In short the offices using a bonus reserve valuation are adopting a similar type of compromise to those using a net premium valuation. Their instrument is in many ways more flexible: while they may find it harder to decide what the 'true and fair' surplus for the year should be, they probably find it a good deal easier to present their results in such a way that the right surplus emerges and that at the same time the company's solvency can be demonstrated.

The view is sometimes expressed that a net premium valuation is a convenient form of presentation of results, but that to find the truth an internal bonus reserve valuation should be made. Paradoxically, perhaps, I believe the exact reverse to be the case: given a thorough analysis of surplus on a reasonable net premium basis, the anatomy of a life fund is clear enough and a bonus reserve valuation, if it gives a different answer (which it frequently does), calls for further investigation. But as a means of presentation of results a gross premium valuation has undisputed advantages and, as will be seen in the next section, may in certain circumstances be imperative.

11. EXTREME VARIATIONS IN CONDITIONS

It remains to consider what is to be done when the swing in the rate of interest is so large that the available buffers are insufficient. It is clear that the basis of valuation of liabilities may have to be changed, but this raises a number of subsidiary problems.

Let us consider a sharp rise in the rate of interest and a fall in market values. If the office has adopted a sound matching policy and has not been careless in granting options, its fundamental position should not be greatly affected. But funds have to be found out of the fall in the value of liabilities to support the assets.

In the first place, it is to be noted that, theoretically, such support for the assets from the liabilities should be a loan and not a gift. This can be seen from consideration of either the liabilities or the assets. If the support is used to write down redeemable assets below their par value, the time will come, as securities approach maturity, when the support is no longer needed and has become asset-surplus.

In the second place the net premium valuation may fail to meet the situation. As has already been mentioned, a net premium valuation is only partially sensitive to a change in the rate of interest because the change in the rate is accompanied by a change in the net premiums—a technical idiosyncrasy which has no counterpart in the facts. In general, a net premium valuation can be described as about one-third or one-half sensitive to interest changes. If, therefore, the rate of interest rises sufficiently high, the associated fall in the value of assets may be so great that the partial relief from changing the net premium valuation rate of interest will be inadequate. Further relief may be found by using modified reserves and releasing the Zillmer deduction

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or by bringing in contingency funds. Taking the extreme case, however, when, although the office is basically sound, there is still a shortfall in the assets, let us consider the mechanics of what happens. Suppose that before the rise in interest the company was earning $2\frac{1}{2}\%$ net, at which rate its existing and new business could each support a 30*s.* simple reversionary bonus, that, by valuing its with-profit business on a 2% modified net premium basis, the surplus was emerging at the rate required to grant such a bonus, and that the business had been approximately immunized on this basis. Now let us suppose that the net rate of interest rises suddenly to $3\frac{1}{2}\%$, that the premium rates are unchanged and for new business will support 45*s.* bonus at the new rate of interest. For new business a net premium valuation of with-profit business at about $2\frac{3}{4}\%$ would allow the new 45*s.* rate of bonus to emerge. However, this same basis cannot be achieved for existing business, since it would, equally as it does for new business, set up reserves sufficient to support a 45*s.* bonus in the future, and there are only enough assets to support 30*s.* bonus.

But a gross premium valuation is not without similar difficulties. A bonus reserve valuation with 30*s.* future bonus and using $3\frac{1}{2}\%$ interest will give approximately the right answer for the existing business and will release sufficient reserves to meet the fall in assets. But the same basis, if employed for new business, will capitalize into surplus the whole of the 15*s.* additional future bonus which the higher rate of interest will support.

There is no simple answer, as this illustration shows. If the change in the rate of interest were permanent and were immediately known to be permanent, it would be possible, and perhaps right, to close the fund for old business and to start a new fund for future business; but it is a principle of this paper that we cannot know that such changes are permanent.

It is perhaps desirable to admit to ourselves that if there is a sufficiently large break in the rate of interest there is no single valuation basis which will adequately answer the two basic questions: 'is the company solvent?' and 'is surplus emerging satisfactorily?' In such circumstances it becomes necessary to divorce the two questions and to concentrate, in the public presentation of results, on an adequate answer to the solvency question, dealing separately with the question of emergence of surplus by internal investigation.

12. BONUS POLICY

A passive system of valuation strictly on the premium bases would allow the surplus to emerge naturally and equitably. This would, of course, entail innumerable subdivisions of the business in valuation according to premium basis at the date of issue. In practice, however, any reasonable net premium basis will be a fair substitute.

How is this surplus to be apportioned among the various policy-holders? If the office has strictly adopted the immunization theory the mathematical consequences to bonus policy of following the theory through would be both uncompromising and uncomfortable. The implications following from the fact that contracts can be immunized at the rate of interest ruling at the date of issue are entirely foreign to the habits of life assurance in this country. Followed to a logical conclusion they would lead either to day-to-day variations in with-profit premium rates, or to complete fragmentation of the business for bonus purposes.

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However, while matching in a general sense is desirable there are many practical reasons for departing from strict immunization. The sharp edges of the mathematical theory are blurred in practice, and it has to be remembered that the immunization of new business is only possible by reason of the existence of the large fund of existing business.

There is much to be said in equity for extending the insurance principle not only to cover variations within groups of policy-holders but also to cover variations between successive groups.

More important, however, than these considerations is the relationship between office and policy-holder. There is an unwritten understanding with the policy-holder that the bonus declaration should be simple and such as to commend itself to him. Justice must not only be done but must be seen to be done, and there is no doubt that a policy of differentiating bonuses according to the rate of interest ruling at issue would never be understood.

Few British actuaries would differ from the conclusion that the wider grounds for uniformity in bonus declarations override the mathematical grounds for differentiation.

If we decide in favour of uniform bonus (simple or compound), the final question is 'at what rate?' This is a deep question, but the following breakdown of the situation clarifies the picture. (The figures are purely hypothetical and explanations follow.)

Source of surplus	Rate of bonus: Now	10 years time	20 years time
A. Bonus emerging under existing business (excluding interest on the estate)	18*s.*	21*s.*	20*s.*
B. Bonus earned by new business at current rates of premiums	23*s.*	24*s.*	25*s.*
C. Estimated effect of amalgamating A and B	18*s.*	22*s.*	23*s.*
D. Interest on the estate then existing, spread as a rate of bonus over the estimated business in force	7*s.*	5*s.*	4*s.*
E. Total bonus coming under review = C + D	25*s.*	27*s.*	27*s.*
F. Contributions from current surplus to estate as planned	4*s.*	4*s.*	3*s.*
G. Declarable bonus = E − F	21*s.*	23*s.*	24*s.*

Notes.

(1) The figures are based on anticipated future experience. They are, of course, highly speculative in the future, but one must do one's best.

(2) The hypothetical variations in A may be due for example to anticipated mortality improvement or to past changes in premium rates.

(3) Item C is based on estimated quantities of future new business.

(4) Item F assumes that the office has some plan for controlling its estate. If it adopts an unmodified net premium basis of valuation there may be a substantial automatic contribution to the estate each year from the net new business strain.

The analysis is illustrative of a technique and the figures are not to be interpreted as a suggested code of conduct.

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13. CONCLUSION

It is fitting perhaps that this lengthy ramble through several aspects of current actuarial problems should terminate on an inconclusive note. It is perhaps the strength of our profession that it is both art and science, and the later stages of this paper have entered those fields where the synthesis of wide judgment is a better guide than the analysis of narrow mathematics (and where the natural desire of an author to be comprehensive may lead him to over-simplify). It is well, however, that we should occasionally pause in the exigencies of our practical routines to consider fundamentals, and that has been the main purpose of this paper.

It remains only to express my acknowledgments. In the main they must go to the profession as a whole, and to the office for which I work, both of which have a corporate mind to which any of their members is a profound debtor. Much of what is contained in this paper has been said before in our proceedings; in particular, I must mention Elderton and Perks (1948, *J.I.A.* LXXIV, 203) and hesitate to do so only because the mention of these names leaves an uncomfortable sense of the names which are omitted. The immunization theory contained in the paper is, I believe, original, and it is important therefore to state that, though they used different approaches, at least two members of the office for which I work reached independently similar general conclusions before the writing of this paper forced my own thoughts into precision, namely, W. E. H. Hickox and P. E. Moody. I should also mention that comments by A. T. Haynes (1945, *J.I.A.* LXXIII, 63) and by Haynes and Kirton (1944, *T.F.A.* XVII, 165–6) while general in form are clearly directed to the same objective as immunization.

Finally, I must thank R. S. Skerman for constant companionship in the work of this paper, and many colleagues—too many to mention individually—for helpful comments.

APPENDIX I

EXAMPLES OF IMMUNIZATION

Table 1 on p. 311 gives a few thumb-nail sketches of typical funds. Some comments on these examples will underline the main points.

(i) In all the examples Rule 1 has been observed, viz. the mean terms of the values of the liability-outgo and of the asset-proceeds have been made the same. The smallness of the profits or losses ensuing on a change in the rate of interest demonstrates the efficacy of this rule as a protection against interest changes.

(ii) Since the liability in example A 1 is a single payment, the asset-proceeds are necessarily more widely spread and a small profit is shown, as Rule 2 implies, whether the interest rises or falls. The amount is, however, very slight. In A 2 the liability is the same but the assets are more widely spread still and the profit is larger.

The only case to show a loss is B 1, where the spread of liabilities is greater than the assets. The loss is very small but could be larger for a real fund with liabilities for all terms.

(iii) The C and D examples are more typical of active life-assurance funds, since the liability-outgo is negative in the early years. The effect of this in

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Table 1. Examples of immunization

Example	Pattern of liability-outgo (1)	Present value at 2½% of liability-outgo (2)	Mean term of the value of the liability-outgo (and of asset-proceeds) in years (3)	Typical immunized asset-maturity pattern using 2½% interest (4)	Profit (per cent. of present value of liabilities) from change in interest rate of	
					+½% (5)	−½% (6)
		£			%	%
A1	£100 after 10 years	78·120	10	£78·120 after 11·485 years	·012	·012
A2	Ditto	78·120	10	£39·060 after 5 years £39·060 after 19·210 years	·054	·059
B1	£50 after 5 years £50 after 10 years	83·253	7·346	£83·253 after 8·106 years	−·005	−·006
B2	£100 after 7·346 years	83·253	7·346	£83·253 after 8·106 years	·005	·005
C1	−£100 after 5 years £200 after 10 years	67·854	16·513	£67·854 after 21·214 years	·137	·155
C2	Ditto	67·854	16·513	£33·927 after 5 years £33·927 after 48·672 years	·396	·485
D1	−£30 after 5 years £60 after 15 years £50 after 25 years £20 after 35 years	50·309	28·982	£50·309 after 50·926 years	·485	·626
D2	Ditto	50·309	28·982	£25·155 after 40 years £25·155 after 65·937 years	·529	·696
D3	Ditto	50·309	28·982	£25·155 after 22·856 years £25·155 after ∞ years	1·022	1·780

Note. Investments are for convenience assumed to be made in stock purchased and redeemable at par under which interest at 2½% per annum is payable continuously, but the principles are the same if stock is purchased at different prices and with different running yields; investments could be made in savings certificates (with no income) or in annuities (with no final capital payments).

lengthening the mean term for the value of the liability-outgo is noteworthy. In particular, the D examples, which can be taken as representative of the funds of most British offices at the present time, show a long mean term for the value of the liability-outgo and consequently a very long mean asset-maturity term (50 years).

(iv) The negative liabilities in the C and D examples not only greatly lengthen the mean term but they are equivalent to narrowing the spread of the liabilities. Indeed, investigation shows that for both these examples the variance, if such a term is permissible in this context, is negative and the standard deviation imaginary. Consequently the second derivative of A substantially exceeds that of L and Rule 2 is obeyed.

(v) The present values in the examples are calculated on $2\frac{1}{2}\%$ interest. If the rate changes, the equations will be slightly different and the weighted mean terms will alter, but not greatly. For example, in D 1 the mean term of the revised value of the liability-outgo at 3% interest is 29·666 years.

In moving from col. (3) to col. (4) however the asset-maturity dates are more sensitive to the proportion of income to capital. In D 1 (keeping to the example with present values calculated at $2\frac{1}{2}\%$ interest as shown in the schedule) the liability-outgo could be immunized,

(*a*) by £43·885 nominal of 3% stock redeemable at par in 54·997 years, or
(*b*) by £58·163 nominal of 2% stock redeemable at par in 46·388 years.

Both these stocks have the required market value at $2\frac{1}{2}\%$ interest of £50·309 and a mean term for the $2\frac{1}{2}\%$ value of the proceeds of 28·982 years.

Another interesting illustration is to follow a single block of policies through their career. The salient points are shown in Tables 2 and 3 on p. 313, illustrating whole-life and endowment assurances respectively.

It is found that there is no real solution for the asset-maturity dates in the early durations of the block. It is obvious that no investment of the first premium in an interest-bearing asset can protect the fund against future changes in the rate of interest. In general, it is found that for a block of new business considered in isolation it cannot be immunized for about the first quarter of the term for endowment assurances or for about a third of the expectation of life for whole-life assurances. At these turning-points immunization requires that the whole of the assets be irredeemable. As the duration of the block of business advances the mean term reduces until finally the assets would all be invested for redemption at maturity.

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Table 2. Working of immunization over the duration of £100 non-profit whole-life assurance with continuous premiums effected at age 25

Duration in force in years t (1)	Present value of liabilities before change $100_t\overline{V}(\overline{A}_x)$ at A 1924–29 ult. $2\frac{1}{2}\%$ (2)	Mean term (from t) of the value of the liability-outgo (and of asset-income) in years (3)	Immunized asset-maturity term in years (from t) if all assets mature on same date (4)	Profit (per cent. of present value of liabilities) from change in interest rate of	
				$+\frac{1}{2}\%$ (5)	$-\frac{1}{2}\%$ (6)
	£			%	%
10	12·325	71·807	No real solution	—	—
20	27·298	34·938	80·417	·784	1·136
30	44·469	20·222	28·017	·036	·040
40	61·920	11·747	13·874	−·034	−·039
50	76·602	6·582	7·183	−·023	−·025
60	86·381	3·679	3·857	−·009	−·009
70	92·044	2·118	2·176	−·001	−·003

Table 3. Working of immunization over the duration of £100 non-profit 20-year endowment assurance with continuous premiums effected at age 25

Duration in force in years t (1)	Present value of liabilities before change $100_t\overline{V}(\overline{A}_{x\overline{20}|})$ at A 1924–29 ult. $2\frac{1}{2}\%$ (2)	Mean term (from t) of the value of the liability-outgo (and of asset-income) in years (3)	Immunized asset-maturity term in years (from t) if all assets mature on same date (4)	Profit (per cent. of present value of liabilities) from change in interest rate of	
				$+\frac{1}{2}\%$ (5)	$-\frac{1}{2}\%$ (6)
	£			%	%
3	11·834	56·420	No real solution	—	—
5	20·271	33·151	69·130	1·135	1·500
10	43·411	13·873	16·985	·076	·088
15	69·778	5·616	6·045	·004	·006

Note. In Tables 2 and 3, investments are assumed to be made in stock purchased and redeemable at par under which interest at $2\frac{1}{2}\%$ per annum is payable continuously.

APPENDIX II

EMERGENCE OF BONUS ON DIFFERENT VALUATION BASES

The table on p. 315 illustrates the effect on bonus earnings of differences between the premium basis and the valuation basis. It must be stressed that these examples are purely illustrative of the consequences of such differences and are not intended to imply that the valuation bases given would be appropriate for practical use. A group of policies issued simultaneously has been examined throughout its lifetime.

Data. The contracts examined are 25-year with-profit endowment assurances effected at exact age 35 subject to an annual premium calculated on the following basis:

Mortality:	A1924–29 ultimate.
Income tax:	7*s.* 6*d.* in £ deducted from both interest and expenses separately.
Interest:	£3. 12*s.* 0*d.*% gross (2¼% net).
Expense loadings:	New business 3·25% (2·03125% net) of sum assured. 4% (2·5% net) of premium.
	Renewal ·25% (·15625% net) of sum assured. 4% (2·5% net) of premium.
Bonus loading:	Provision for 20*s.*% simple reversionary bonuses vesting at the end of each policy year.
Contingency loading:	Nil.

The premium rate is ·04343092 per unit sum assured.

Assumptions

(1) That the experience as regards mortality, tax, interest and expenses, is that assumed in the premium basis.

(2) That claims are paid at the end of the year of death or on survival to maturity and that no withdrawals occur.

(3) That the whole of the surplus emerging in any year (whether positive or negative) is immediately distributed as bonus.

(4) That the valuation basis is always based on A1924–29 ultimate mortality.

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Table 4. Rate of bonus per cent. which emerges in the assumed conditions according to various bases of valuation

Example	Valuation basis	Rate of bonus per £100 emerging in year						
		1	2	5	10	15	20	25
(*a*) Unmodified net premium	(i) 2¾ % interest	−32*s.* 4*d.*	37*s.* 3*d.*	32*s.* 3*d.*	25*s.* 6*d.*	19*s.* 6*d.*	14*s.* 1*d.*	9*s.* 2*d.*
	(ii) 2½ % interest	−34*s.* 11*d.*	27*s.* 11*d.*	26*s.* 2*d.*	23*s.* 8*d.*	21*s.* 5*d.*	19*s.* 4*d.*	17*s.* 7*d.*
	(iii) 2 % interest	−36*s.* 0*d.*	23*s.* 8*d.*	23*s.* 4*d.*	22*s.* 9*d.*	22*s.* 4*d.*	21*s.* 11*d.*	21*s.* 8*d.*
(*b*) Modified net premium	(i) 2¾ % interest	34*s.* 6*d.*	33*s.* 1*d.*	28*s.* 10*d.*	22*s.* 3*d.*	16*s.* 6*d.*	11*s.* 7*d.*	7*s.* 0*d.*
	(ii) 2½ % interest	24*s.* 11*d.*	24*s.* 5*d.*	23*s.* 0*d.*	20*s.* 11*d.*	18*s.* 10*d.*	17*s.* 0*d.*	15*s.* 5*d.*
	(iii) 2 % interest	20*s.* 8*d.*	20*s.* 7*d.*	20*s.* 4*d.*	20*s.* 1*d.*	19*s.* 10*d.*	19*s.* 8*d.*	19*s.* 7*d.*
(*c*) Unmodified bonus reserve	2½ % interest 20*s.* % future bonus	−63*s.* 7*d.*	24*s.* 9*d.*	24*s.* 6*d.*	24*s.* 1*d.*	23*s.* 8*d.*	23*s.* 3*d.*	23*s.* 0*d.*
(*d*) Modified bonus reserve	(i) 2½ % interest 20*s.* % future bonus	16*s.* 2*d.*	20*s.* 3*d.*	20*s.* 2*d.*	20*s.* 2*d.*	20*s.* 2*d.*	20*s.* 2*d.*	20*s.* 2*d.*
	(ii) 3 % interest 20*s.* % future bonus	252*s.* 8*d.*	18*s.* 6*d.*	15*s.* 7*d.*	11*s.* 3*d.*	7*s.* 5*d.*	4*s.* 3*d.*	1*s.* 9*d.*
	(iii) 3 % interest 30*s.* % future bonus	34*s.* 10*d.*	28*s.* 6*d.*	25*s.* 6*d.*	21*s.* 3*d.*	17*s.* 4*d.*	14*s.* 1*d.*	11*s.* 6*d.*

Notes. (1) In (*b*) a Zillmer rate was used corresponding to the additional new business expenses in the premium basis (i.e. 3 % gross = 1·875 % net).

(2) The expense ratios used in (*c*) and (*d*) were 15 % gross (9·375 % net) over-all and 10 % gross (6·25 % net) renewal respectively, and as such are crude practical approximations to the premium basis.

ABSTRACT OF THE DISCUSSION

Mr F. M. Redington, in introducing his paper, referred to the paper by Messrs Haynes and Kirton which had been submitted to the Faculty in the previous month and which he hoped members of the Institute would read. Both they and he had, at a late stage, become aware that they were writing papers on similar subjects, but they had deliberately avoided discussion, which would have been embarrassing to both, during the preparation of the papers. Their paper, he thought it fair to say, dealt primarily with matching, valuation being a by-product; his own paper dealt primarily with valuation, matching being a by-product. It was remarkable how closely they agreed in their fundamental conclusions.

He wished to remove any misunderstanding that might have been left by his reference to a bonus reserve valuation. In his opinion the bonus reserve valuation was an exceedingly useful practical instrument. In certain extreme conditions of high or low interest rates offices might be compelled to depart from the net premium valuation and to adopt a bonus reserve valuation. Speaking for himself he would be quite happy to do so. His objections arose when the bonus reserve valuation was put forward as the proper method of valuation. He could not accept such a view, if only for the simple and, it seemed to him, obvious reason that there was no basic principle by which a uniform future bonus should be expected. The assumption of uniformity evaded the whole theoretical issue.

Mr G. V. Bayley, in opening the discussion, said that the author had, in his own words, taken the reader on a ramble through the actuarial countryside. The journey fell naturally into two parts, the first being concerned with the theory of matching assets and liabilities. Immunization was defined on p. 289 as

> the investment of the assets in such a way that the existing business is immune to a general change in the rate of interest.

It was perfectly general, and as it stood it did not attempt to provide for a marching forward of events. It was concerned only with the position on a certain date. That was simply illustrated by the numerical example on p. 291, where a single payment at the end of ten years was matched by a $2\frac{1}{2}\%$ stock redeemable at the end of 11·485 years. That was only correct at the time. A year later, the figure moved closer to 10 by ·307 years, and the original investment had to be changed in order to obtain the match.

The definition of immunization led to an infinite variety of solutions to equations (1) and (2) on p. 290. Many, though not all, of those solutions demanded substantial realization and reinvestment of funds from time to time. He did not want to exaggerate their magnitude, but wholesale changes of the portfolio were, of course, costly and often impracticable, so some constraint in that direction was imposed in practice. Suppose, therefore, that the definition of immunization was qualified by the requirement that there should be minimum realization and reinvestment as time passed. For all types of liability-outgo he had thought of which admitted real solutions, that simple requirement had the effect of defining precisely the asset pattern. In other words, it selected from the infinite number of solutions available a unique solution in every case. For example, for decreasing funds the solution was given on p. 292 at (*f*). The author called it 'absolute' matching, and no subsequent changes of investment

were required in that case. For the 10-year unit liability on p. 291, if the assets consisted of 2½% interest-bearing stocks, 96·4% would have to be redeemable at the end of 10 years and the balance irredeemable. For funds which had not reached their peak, the solution would be similar and would require the holding of a proportion of irredeemable stocks to compensate for the investments that had to be made during the next few years. As might be expected, the unique solution in that case became an absolute match without discontinuity at the point of time when the fund became a maximum.

It was worth selecting that particular solution, not simply as a theoretical exercise, but because it defined asset patterns similar to those which would be encountered in practice or, indeed, chosen as a practical objective.

It was interesting to notice that both the general and the unique solution depended upon i, the current rate of interest, and g, the coupon rate. For example, if g were low, there would be less investment for an increasing fund during the early years, and the vulnerability of its fresh investments to changes in the rate of interest would thereby be lessened.

The application of immunization in practice must, he thought, have regard to the laminated character of the fund. For example, recent new business on its own could not be immunized, and he would like to call attention to the practical consequences of aggregating all durations for the purpose of immunization. When rates of interest rose, recent entrants might find it profitable to lapse and re-enter or, worse still, to go to another office. That suggested to him a cardinal principle for with-profit business: to the extent that the fund contained participating policies, it should only be immunized on the basis of the premiums paid to date. In that way, the bonus-earning power of existing business ran smoothly into that of new business. Such an approach led to principles of bonus distribution and immunization which could be formulated as precisely as any that flowed from an attempt to immunize the fund at an existing rate of bonus. It led to asset-patterns based on paid-up policies and therefore rather shorter mean terms than for total immunization. In strict equity, the principle would also lead to a variation in bonus according to duration, and violent changes might justify that course: rough justice might be achieved in many circumstances by a gradual change from the old bonus level to the new.

Looking at the question in another way, the right to pay with-profit premiums over a period of years seemed to lead logically to a bonus system which recognized the variation in rates of interest over that period, and not simply the rate ruling at the inception of the contract. Partial, or paid-up immunization as one might call it, gave ideal expression to those conceptions of equity. Total immunization seemed to reduce the character of a with-profit policy to that of non-profit and it was questionable whether total immunization was not over-immunization for participating business. To the extent of the difference between total and paid-up immunization, it seemed to him that the office was taking a view. That might be legitimate—indeed, circumstances might make it essential—but the author had given a warning of the consequences throughout his paper, and in particular on pp. 293 and 294.

The remainder of the paper was devoted to valuation strategy in general and in particular. Fundamental to the author's consideration of the net premium method was his conception of the office's estate. It was a helpful idea in the rationale of that method, but if a bonus reserve valuation was performed, it had to be remembered that a large part, and in certain circumstances the whole, of the estate, as defined on p. 296, comprised the reserve for future bonuses. For

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example, the conception of a capital transfer to or from the estate due, say, to a change in the mortality basis did not take quite the same form. In a net premium valuation a capital loss transferred to the estate had the practical effect of first a permanent annual reduction in surplus equal to interest on that loss, and secondly a readjustment of the annual sources of surplus. In a bonus reserve valuation, the cost of a change of that nature might be borne at more than one stage. For the purposes of the immediate declaration, the cost at any stage had to be apportioned against the immediate distribution and against the reserve for future bonuses. At subsequent valuations, the actuary had the freedom to dispose of the unrequited balance of cost in a similar manner.

Offices publishing the results of a bonus reserve valuation frequently retained certain margins, but for internal purposes a line could be drawn at any point between a provision for future bonus of, say, 52*s*., and a provision for future bonus of 35*s*. plus an implicit or explicit estate. The essence of the method was that there was greater freedom of manoeuvre, and the direct expression given to the two variables, present and future rates of bonus, should help to achieve equity between different generations of policy holders.

With regard to the concept of natural revenue surplus, the author said on p. 302

> The great justification for the net premium valuation is that it gives an approximation to the premium basis of valuation and therefore to the natural revenue surplus.

An attachment to natural revenue surplus would define a particular rate of release of surplus per policy, and a glance at Table 4 showed that that might not lead to level reversionary bonuses. The approximation to the premium basis might be only rough, and it was more a lucky consequence of the method than a reason for it. The margin between the rate of interest earned on the fund and that used in the premium basis might not be sufficient to hold back surplus, especially if the premium were loaded for a high level of reversionary bonus. Natural revenue surplus might therefore lead to falling bonuses. Surely the great justification for the net premium valuation at an appropriately low rate of interest was that it released surplus at about the right rate to support level reversionary bonuses.

Natural revenue surplus was stated on p. 301 to be the only clear conception of surplus, but there was one other: a gross premium valuation using the best possible estimate of future experience produced surplus which was the best estimate of the present value of past and future profit, less, of course, any amounts already distributed and assuming that no further new business was transacted. That was, he submitted, a clear conception and a clear basis for a distribution by the bonus reserve method. That it capitalized any difference between the valuation and premium bases was not a difficulty but a *sine qua non* and it corresponded directly to a capitalization of future bonuses on the other side of the balance-sheet. The real difficulty was one of apportionment of total profit, including capitalized future profit, between past distributions, general reserves, present bonuses and future bonuses, of which only the first was fixed. In exercising judgment upon that apportionment, the indispensable documents referred to at the bottom of p. 302 became instead (*a*) an analysis of surplus, including the cost of any change in the valuation basis; (*b*) an investigation of trends and miscellaneous sources of profit; and (*c*) an investigation of the bonus-earning power of new business. Admittedly, an error of judgment might lead to an over-generous distribution at a given time, but so also would a net

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premium valuation which failed, for example, to take proper account of a short position in the assets when rates of interest had recently fallen. He therefore supported the author's suggestion, in the case of a net premium valuation, of a statement regarding the matching of investments against liabilities. If a bonus reserve valuation were performed on a realistic basis, it was more sensitive to such a 'mis-match' and the need for the information was less compelling.

There was obviously much to be said for and against the two methods of valuation. He would, however, confine himself to a brief reference to three features of the bonus reserve method. First, if the bonus system demanded the release of surplus at a special rate, there was a particular need for that method. Secondly, it was obviously difficult to capitalize all sources of future profit applicable to and during the lifetime of existing policy-holders (if, of course, it was desired to do so). Thirdly, he agreed with the author that it was logical to bring in assets at market value. If any other course were pursued, the choice of the valuation rate of interest would be much more complicated.

The two- or three-cushion system of reserves was algebraically equivalent to a single-cushion system. Assuming assets and liabilities to be well matched, which was the author's own qualification, there seemed to be no clear meaning to the statement on p. 305 that the fund could stand a rise or fall of $(x+y)\%$ in the rate of interest. In the assumed conditions it would stand a rise or fall of much more. A single central reserve in matched conditions had a tangible meaning only if it was expressed simply as a sum of money, or more imaginatively, as being able to support a future bonus at $z\%$. That clearly brought the author full circle to a plea for a bonus reserve method of valuation!

Mr C. D. Rich said that he was particularly interested in the theory of 'immunization', to use the name which the author had bestowed upon it—though he wished that a different name had been chosen, for the word 'immunization' sometimes had unpleasant associations. He would like, instead, to suggest the word 'conjugation'—in the sense of 'yoking together'. A Past President of the Institute had said, with reference to the redemption dates of the investments, that a life office should 'marry its assets to its liabilities'; it was with that idea in his mind that he suggested the phrase 'conjugating the assets and the liabilities'.

Immunization, or conjugation, was an outstanding example of the difference between actuarial theory and practice. How delightful it would be if the funds of a life office could be so invested that, on any change in the rate of interest—whether up or down—a profit would always emerge! But how difficult it would be to carry out to the full the investment policy implied by the theory of immunization.

The assets and liabilities were conjugated if their first differential coefficients with respect to the rate of interest were equal. As the opener had stated, there was in general an infinite number of ways of arranging the redemption dates of the assets to satisfy this condition. Assuming that the investments were all of present value equal to their redemption values, bearing nominal rates of interest equal to the valuation rate, two distributions of the redemption dates were of particular interest, namely (1) when all the investments were redeemable on the same date (which might be called the 'conjugate' date for the liabilities under consideration), and (2) when the investments were divided in appropriate proportion between what Haynes and Kirton had called 'dead short' and 'dead long', i.e. between money on deposit and irredeemable securities. Those two

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distributions were the ones giving respectively the smallest and the greatest second differential coefficients to the value of the assets, i.e. of all the conjugate distributions they would produce the smallest and the greatest profits on a change in the rate of interest.

A distribution of the investments which was more akin to distributions that arose in practice, and which lent itself to analytical treatment, could conveniently be obtained by supposing the terms to maturity of the investments to be distributed continuously from 0 to infinity, the amount invested in securities of unexpired term n being proportionate to $1/(1+k)^n$, where the value of k was fixed by the necessity of making the distribution of the investments conjugate with that of the liabilities, i.e. dependent on the mean term of the liabilities. Such a distribution might be called a 'geometrical' distribution, since it was similar in character to a decreasing geometrical progression.

The problem of immunization was essentially one of compound interest. Mortality did not really enter into it, and its general investigation could be considerably simplified by examining the position of an office transacting not life business but capital redemption business. It was, for example, comparatively simple to construct a model office whose business consisted of, say, 10-year sinking-fund policies by annual premiums, or of 20-year policies, and so on. For convenience, continuous functions could be used, and a single formula could be developed which would represent the reserves of a stationary office, or of a uniformly increasing office, or of a uniformly decreasing office.

Some time previously he had made some calculations using a basic rate of interest of $2\frac{1}{2}\%$ per annum, it being assumed that both liabilities and assets were valued at that rate and that the same rate was used for the calculation of premiums. He had found that in the case of a uniformly increasing office, increasing at the rate of $2\frac{1}{2}\%$ per annum, and issuing 30-year sinking-fund policies, the fund could be invested at $2\frac{1}{2}\%$ so as to be conjugate to the liabilities in the following ways:

(1) all in securities redeemable at the end of 26·4 years (the conjugate term),
or (2) 52·1 % on deposit and 47·9 % in irredeemable securities,
or (3) in a 'geometrical' distribution with the terms to redemption running from 0 to infinity, the value of k being ·0272.

The results of a fall in the rate of interest to 2 % or 1 %, or of a rise to 3 % or 4 %, on the value of the liabilities and on the value of the assets according to the three methods of investment were:

Rate of interest (%)	Value of liabilities	Value of assets		
		(1)	(2)	(3)
1	1339	1343	1710	1401
2	1100	1101	1118	1105
$2\frac{1}{2}$	1000	1000	1000	1000
3	910	911	921	914
4	758	761	822	780

If negative values were excluded, the value of the liabilities at 3 % and 4 % became 914 and 782 respectively. The close correspondence of those figures to the value of the assets when invested in a geometrical distribution was noticeable. In fact, it was apparent that such a distribution of the investments should give a satisfactory result however the rate of interest might move.

It could be shown that the greatest profits on a change in the rate of interest (assuming that the assets were conjugated with the liabilities) would arise in cases where the conjugate term was long, i.e. if the fund was young or increasing rapidly. If, however, the fund was very young or increasing very rapidly, the point was reached at which the plunge was taken beyond the infinite to the imaginary, and after that point there was no real solution for the value of the conjugate term. It might, in fact, then be said that the solution involved, not i = the rate of interest, but i = the square root of -1.

An interesting question was what was the rate at which an office should uniformly increase in order that the conjugate term should be infinite, i.e. so that its assets when correctly conjugated with its liabilities would consist entirely of irredeemable securities. An office with such a rate of increase (which might be called the 'critical' rate of increase) obtained the maximum profit on a change in the rate of interest; but an office increasing at a rate more rapid than the critical rate could not possibly conjugate its assets with its liabilities—except by the artifice of borrowing 'short' and investing 'long'. In the case of an office issuing 30-year sinking-fund policies, the rate of interest being $2\frac{1}{2}\%$, the critical rate of increase was 12·4% per annum.

One lesson which should be learnt from the theory of 'immunization' or 'conjugation', and which was borne out by some of the examples in Table 1, was that the investments of a life office should in general be in securities of much longer term than many people realized. That applied more especially to a young or increasing fund, i.e. to a fund possessing the characteristic 'hump' mentioned by Messrs Haynes and Kirton. It could in fact be said that whereas an office could rely upon earning the current rate of interest upon its existing fund, there was a risk of its not being able to do so upon future increases in the fund, and that the risk could be covered by investing the existing fund in securities of mean term longer than the mean term of the liabilities.

Mr H. F. Fisher approached the subject from the point of view of practical problems arising in the valuations of collecting friendly societies. Such valuations, especially of the larger societies, were not primarily for solvency purposes; and the problem of dealing with the assets assumed increased significance when it was remembered that the investments were limited to trustee securities. It might be of interest to state that the largest collecting friendly society held assets of more than £111 million at the end of 1951, of which some £48½ million were invested in British Government and other securities, the market values of which had fluctuated considerably.

There was a slight difference in the statutory duty of the actuary in valuation. Under Section 28 of the 1896 Act, the valuer was required to value both assets and liabilities, whereas under the 1909 Act the actuary's certificate, at all events, related only to liabilities. A comparison of statutory balance sheets showed that Form C. 28 of the 1896 Act invited the valuer to deal with depreciation in spite of the certificate of the officers of a society as to the value of the assets.

There were two points in the immunization idea that rather appalled him at first glance. The first was that by stabilizing the bonus of existing entrants at a low rate it was possible also to stabilize the bonus of new entrants at a higher rate, and the opener had put his finger upon that by suggesting a limited immunization. Although the author did not suggest that such a policy should be followed, but had merely discussed it, he had shown that a danger existed. The second point was the idea of immunizing a fund against making a profit.

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The structure of the author's immunization theory itself arose on the basis that V_A' varied from V_A by the same amount as V_L', the value of the liabilities, varied from V_L, and it accepted implicitly a market revaluation of existing assets on a change in the market rate of interest. On p. 304, however, in the second paragraph, the author suggested that while that was necessary for a solvency valuation, a passive valuation policy for both assets and liabilities was fairer when considering a distribution of surplus or equity valuation. But further down the page the author came down rather heavily in favour of market values on the ground of the public understanding. Again, when dealing with bonus policy on p. 308, he suggested that the passive or net premium valuation basis was necessary for the liabilities, at the same time implying that the active or market valuation basis was the correct one for the assets.

In the light of that, the speaker presented the problem of the actuary of a collecting society, holding British Government securities, the market rate of interest having risen by, say, 1 % since purchase. If those dated securities in the portfolio were reasonably matched, the market price obtainable was the price obtainable by a willing seller; but if they had been bought to hold, as was usually the case, the society was an unwilling seller. As the actuary had to value assets and liabilities, on the one side he valued the promise of the society to pay the sum assured less the promise of the member to pay the premiums; should he not value on the other side the British Government's promise to pay the interest and capital at the appropriate times? He would say—perhaps with his tongue slightly in his cheek—that accepting the market value in those circumstances was like valuing the liabilities by putting policies up for public auction. At all events, the actuary must not, in valuing the promise of the Government to pay the interest and capital, use a rate of interest which in any way implied that he was taking credit for any part of what the author termed the estate. In other words, if the valuation rate of interest on the liabilities had been artificially depressed in order to produce part of the estate within the liabilities, such a rate of interest would obviously not be the correct one for valuing the assets. On the other hand, where the assets were dated and matched, to take the other extreme of market values was not in keeping with the true picture. Further, it might be that those securities were all purchased and held in the books at less than par. In such circumstances, there was a margin, which would be realized on redemption at par, between the book values, which could be termed the notional values, and the capital which would actually be received.

The same remarks could not also be true of irredeemable or undated securities, and if the same view were taken regarding the valuation rate of interest for assets it would be tantamount to assuming a continuing new business. The valuation of assets must clearly have regard only to policies on the books, but market values nevertheless might be unreasonably low. If part of the high yield obtainable on undated securities was regarded as a risk premium for capital loss and if, as a matter of investment policy, values were written down each year, it might be that the book values were in any event lower than the market values; the inclusion of the investment reserve fund might act as the other cushion.

The method of provision for the expenses of a collecting society out of premium income—usually a percentage of premium—made a gross premium form of valuation particularly suitable in the industrial branch, so long as the amount set aside for management was adequate. In the ordinary branch, where the question was, perhaps, more one of equity and a stable bonus policy than of

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solvency, there arose the problem with which the author had dealt. Was it to be solvency and an active policy or equity and a passive policy?

He shared the opener's opinion that the author had made out a case for a bonus reserve valuation in spite of himself. The speaker did not understand the paradox, but still thought the net premium valuation was the right one. A bonus reserve valuation which valued that part of the bonus which had been allowed for in the bonus loading of the premiums, combined with a form of premium properly unloaded for expenses, might be halfway between the two, and certainly for the purposes of a collecting society would be a stable one. It would clearly not be right to include in the rate of future bonus that part which the author had shown truly belonged to the estate. Nevertheless, that part which was to be allowed for in the premium loading was appropriate to appear in the bonus reserve valuation. It might be that with the established practice of profit-sharing in the industrial branch, at some time the bonus reserve valuation could also be published for industrial business, particularly if the premiums were loaded, or considered to be loaded, for bonus.

To return to the statutory forms and certificates regarding assets, the statutory forms referred to value of assets and not market value. It might be of interest to point out that because of the difficulties of some of the collecting societies on 31 December 1951, the Chief Registrar had agreed, for the year 1951, to the deletion by all collecting societies of the certificate regarding the value of assets which was normally required in form A.R. 10. There was no statutory authority for that certificate, except in the year of valuation when it was required to be given.

The legislation did not provide for the use of market values, and in certain circumstances the actuary should be free to place upon the assets the values which in all the circumstances were appropriate to the case.

Mr R. J. Kirton thought that it did not matter whether an approach were made to the author's immunization theory on the assumption of a uniform rate of interest and uniform changes in that rate after relegating varying yields and varying differentials to the realm of practical complications, or whether, on the other hand, the idea were developed against a generalized background of a varying interest structure with the uniform rate treated as a special case.

The mathematics resulting from the uniform rate assumption were fascinating and the results startling. What did seem to be of the greatest possible importance was the underlying theory, because without such an appreciation it was impossible to tell at any moment of time whether the assets of a life office were indeed long or short relative to their liabilities. It was important to realize that the immunized distribution was not one to be followed slavishly. Rather, it was a yard-stick from which departure was made in the exercise of normal investment judgment, the departure being justified by the existence of free reserves sufficient to cover any loss, should that judgment be faulty.

A second point was that embodied in the author's paper under the title of 'estate'. That too seemed to be an idea of great importance in appreciating the position of a life fund, namely that there existed or should exist in one way or another a body of free reserves carrying interest. Those free reserves—the estate—were augmented from time to time, drawn on from time to time, or interchanged among themselves. They formed the cushion which, through the major fluctuations that would inevitably occur, either in taxation, mortality, interest or expenses, should protect the life fund. He strongly supported the

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author's plea for flexibility in the methods by which those reserves were held.

One point of importance which he missed in the paper was the idea that the existence of a proper amount of with-profit business formed one of the most important cushions in the structure of a life assurance fund.

He was interested to see that running through the paper was linked thought on assets and liabilities—an appreciation of their interrelationship. Such thought and such appreciation would seem to lie at the very foundation of the actuary's problems and were vital to the proper discharge of his professional responsibilities, in connexion with all such funds, whether they were pension funds or life funds, or long-term funds.

Mr Wilfred Perks described the paper as one after his own heart. It was a nice combination of theoretical principle and practical wisdom, with a valuable piece of new technique thrown in.

The new technique was based upon the old principle of approximating to a product-sum. That was the principle of the n-point method. The author substituted for the distribution of the value of the liability-outgo another distribution, representing the value of the asset-proceeds which had the same mean and standard deviation and possibly some of the higher moments. In that way he obtained close agreement at the new rate of interest, whatever it might be. He worked on the product sum, $\Sigma e^{-th}F_t$, where h was the change in the force of interest and F_t was either the value of the liability-outgo for duration t, or the value of the asset-proceeds for the same duration.

The author referred to immunization at a force of interest δ. It was just as appropriate to immunize at zero interest, taking the moments of L_t and A_t, and there were obvious practical advantages in doing so. In fact, the method was a combination of n-points and n-slabs. The points were the asset-maturity dates, and the slabs were the dividends. It was very pleasing to find that the n-point method had become of interest to the Investment Managers!

With regard to valuations, there were many points of detail in the paper on which he would differ or to which he would give a different emphasis, but in broad principle he went a very long way indeed with the author. He liked particularly the author's *exposé* of the net premium method and of the passive policy of valuation. His own thoughts had long been in the same direction, as his paper of 1933 showed (*J.I.A.* LXIV, 264). He had thought for a long time that students had had far too much of the bonus reserve method pumped into them over the last quarter of a century, and the paper under discussion was a valuable counterblast.

His main difficulty with the paper was in the valuation of the assets. In the discussion on Pegler's paper, he had objected to writing the assets up to market values. He equally objected to having to write the assets down to market values. It involved a liquidity principle which to his mind was irrelevant to a life office. It was significant that the legislature had not required it in 1909 or 1946. He felt that the author had drawn heavily on his imagination when he said that there was 'a public instinct to take account of market values'. It seemed to him to be rather a strange instinct that was content with such a test at an arbitrary and possibly self-selected point of time once in every five years.

There was, of course, a vast difference between a real prospect of loss on an investment and a mere change in the interest rate for marginal deals at a particular point of time. He felt that it did not help to clarify thought to assume

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that those two completely different situations required the same treatment. In his opinion the author confused them when he referred to 'real insolvency arising from badly matched assets'. The trouble was with the concept—he used the word 'concept' very deliberately—of a uniform rate of interest for valuing future payments, particularly when that rate was the rate at which current prices of certain assets could be reproduced. A uniform rate of interest was part of a theoretical model which reflected the practical world up to a point; but the analogy should not be pushed to the extent of assuming that real investment transactions would necessarily take place on that basis at all future points of time. Indeed, it would be wrong to assume that a large portfolio of investments could be bought or sold on that basis, even at the time of valuation.

He had the impression that the author did not always appreciate that the arguments were often very different as between a largely with-profit portfolio and a largely non-profit portfolio of life assurance business. For with-profit portfolios in Great Britain, there could rarely be any doubt about solvency, either actual or apparent. Immunization as discussed by the author might possibly be appropriate for non-profit portfolios, but the consequences for a with-profit portfolio would be fantastic. To fix bonuses according to the rate of interest that happened to apply at the date of issue would be quite unacceptable. The author said that it would never be understood; he would go further and say that it would be idiotic. Perhaps he might be forgiven for using an adjective of that kind, because he found a number of adjectives in the paper which were equally coloured. He might, perhaps, refer to the first paragraph of the *Conclusion*, where there was a reference to wide judgment being a better guide than narrow mathematics!

A with-profits policy-holder expected each of his premiums, less the current expenses and the cost of the death risk, to be invested as remuneratively as possible, and he expected that to be reflected in his own bonuses. A recent policy-holder would not expect to have to boost the assets of the older generation. With those ideas in mind, he had given a little thought to the problem of matching. His own conclusions were tentative, but he was encouraged to mention them at that stage, because he found they were so similar to the conclusions reached by the opener.

For his purpose, he had borrowed some of the ideas of the old re-insurance method of valuation and, like the opener, he had thought in terms of the theoretical paid-up policy at the valuation date. The rough matching of assets to meet those amounts when they matured seemed reasonable. As premiums were paid, the paid-up policy increased, and corresponding assets would have to be acquired. Part of the earlier bonus loadings would have to be saved for the later durations, and that could be done by valuing the theoretical paid-up policies at a somewhat lower rate of interest. The rest of the bonus loadings would normally fall into surplus to provide current bonuses. Otherwise, they would provide a buffer against the possibility of a catastrophic fall in the rate of interest that could be earned on new investments. On such a basis, a rise in the rate of interest would be wholly beneficial, and that was a situation in which he thought most life offices would like to be. At any rate, in that way a sound theory of valuation of assets and liabilities and a consistent system of investment and surplus distribution could be built up which did not run into the logical and practical difficulty of the immunization theory with bonuses tied to the rate of interest prevailing at the date of issue. It also showed the appropriate steps to be taken if, owing to a rise in the rate of interest, an office wished to dress its

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window by writing its assets down to market values and making further book-keeping entries to correspond. It meant, too, that the existing business and the new business would stand on their own separate legs.

In practice, no doubt, offices would invest rather longer than the theory implied, but they would do so at the expense of mixing up the finances of the new and the existing business.

Mr A. F. Murray said that the crucial conception in the treatment of the problem of matching was the inclusion of interest as part of the asset-proceeds. As soon as that step was taken, the problem was considerably simplified and at once became susceptible to a straightforward mathematical treatment. The mathematical analysis showed that, under the ideal conditions assumed, the mean term of the value of the asset-proceeds and of the liability-outgo should be equal if both functions were to be equally sensitive to changes in the rate of interest. Such a solution was to be expected and was, indeed, almost self-evident from the definitions. That did not detract from the value of the conclusion, rather did it emphasize the inherent logic of the definitions that the author had adopted.

What was somewhat unexpected was the second conclusion that a change in the rate of interest could produce a profit for the immunized fund if the spread of the asset-proceeds was wider than the spread of the liability-outgo. How far this could be achieved in practice was doubtful, but it was at any rate a cheering thought for an Investment Manager when an uncooperative Stock Exchange refused to produce securities for a term certified as correct by the actuary. After listening to Mr Perks he looked forward to the additional pleasure of being able to tell the stockbroker that he was not paying proper attention to the n-point system.

Theoretical analysis such as the author had suggested was extremely valuable in so far as it indicated the ideal maturity distribution of the assets under certain clearly defined conditions if the office was to be immunized against fluctuations in the interest rate. Because of the impossibility of realizing the ideal conditions in practice, however, the author rightly issued a warning against applying the theory as a system of investment.

The difficulty was that while a change in the rate of interest affected the values of the liabilities in a precise and easily calculated manner, the effect on the values of the assets was usually blurred and distorted by secondary factors arising from that change in the rate of interest. The existence of wide optional redemption dates was probably the main disturbing feature. Wide option dates existed not only in Government securities but in many debentures and even preference shares. That borrowers should want those option clauses, particularly during periods of relatively high interest rates, was understandable, but their undesirability from the office's point of view was obvious.

He would like to think that a consideration of the paper would impress upon those responsible for the investment of insurance companies' funds the necessity of exercising the greatest discrimination in subscribing for issues carrying wide option dates. It might be argued that to refuse to subscribe for such issues would be unfair to the borrower, but it should be remembered that the main purpose of high interest rates at the time of speaking was to deter borrowers, except in special cases; quite clearly the effect of a high interest rate would be minimized if the borrower were able to invoke an option against the lender at a comparatively early date.

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Another example of the refusal to react to a change in the interest rates to the extent expected theoretically was given by preference shares. If interest rates were low, preference issues carrying a high rate of dividend stood at a lower premium than was warranted by the price level of low dividend preference issues of a similar standing, because of the possibility of circumstances arising which might lead to their redemption at par. In a portfolio carrying a large proportion of such shares the total effect on the value might be considerable.

The paper suggested an interesting line of thought on the suitability of ordinary shares for an insurance company. The office investing in equities presumably expected a steady appreciation in the values of such shares and gradually rising dividends over a long period. If that were accepted, then it followed that ordinary shares were, from the point of view of maturity, 'longer' securities than irredeemable debentures, and the office which found difficulty in obtaining investments sufficiently long-dated would, other things being equal, look with all the more favour on equities.

Similar considerations applied to freehold ground-rents with a large reversionary element in them, but there the practical difficulty arose that the potential insurance investor would be competing in a market which was much more attractive to the sur-tax payer. That was merely another illustration of the difficulties that faced the investor who endeavoured to translate into practice any clear-cut theory on investments.

Mr M. E. Ogborn referred to the textbook by Coe and himself, who were in the same difficulty as the author. They felt that there was nowhere in the literature a precise and accepted statement of principle with regard to the matching of assets. He would support what the opener and others had said: the future could not be foreseen, hence the premiums for participating business should be on such a basis as would in all likely eventualities be sufficient to pay the basic liability and the policy-holder should be given, by way of compensation, a share in surplus. If that principle were accepted, then future premiums need not be immunized; changing conditions would affect future premiums on existing business in the same way as future premiums on new business and immunization need only relate to the part of premiums which had been paid to date.

If the theory of immunization was intended to commend the transaction of non-participating business only, the fund being safeguarded by a policy of immunization, he would be strongly against that course. For the good of the business an adequate proportion of participating business was needed.

The author was strongly of the opinion that the income tax rebate should be deducted from expenses; i.e. the true net interest method rather than the effective net interest method should be used. There were dangers in either method. The choice seemed to be between being logical but inequitable—as in the true net interest method—and being equitable but illogical—as in the effective net interest method. Each method was preferable in some circumstances. If the author would follow out his own suggestions in, for example, the problem of surrender values, he would probably agree that there was more in it than he suggested.

On p. 297 the author referred to the allowance for initial expenses. Personally, he felt that the allowance should be limited to the actual out-of-pocket expenses which varied automatically with the new business; if that were done the author would not have the difficulty he mentioned with renewal expenses. The term

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'renewal expenses' was woolly, and the expenses needed to be divided into three items: initial expenses, renewal expenses and the expenses which, although neither one nor the other, were a necessary part of a continuing portfolio whatever the level of new business being transacted.

On p. 300 the author drew a distinction between capital surplus and revenue surplus. He himself would deny that there was any distinction. For many years there had been a theory that some difference existed between capital and interest on an investment; he thought that the theory had been effectively 'Peglerized' (*J.I.A.* LXXIV, 179). Having said that, he did agree that there was a distinction between surplus that was distributable and surplus that was not, so perhaps they both came to the same conclusions at last.

On pp. 302 and 303 the author praised the net premium method and blamed the bonus reserve method, but he did both for the wrong reasons. He praised the net premium method because it gave an approximation to the natural revenue surplus. The speaker challenged anyone who made a net premium method to say that the valuation showed the natural revenue surplus. It was not true. The basis would need a good deal of adjustment to give that position.

Equally, the comments on the bonus reserve method were beside the point. The alarming table at the end would be a good deal less alarming if account were taken of the fact that a valuation dealt not with one duration only but with a mixture of durations. There were two major difficulties with the bonus reserve method, and they were very real difficulties.

First, it involved a considerable amount of work, and he could speak with feeling, because it had been used in his office for some twenty years. The work fell on the actuary or on his qualified staff, and it fell at an inconvenient time. Though the amount of work was not an objection if the method was essential, it was a real factor to be taken into account in considering the method.

The second difficulty was that the method assumed that the actuary was free to decide policy, as had been mentioned in another way earlier in the discussion. In practice the actuary's hands might be tied—for example, the assets might be valued at conventional book values and, if so, the advantages of the bonus reserve method might be lost.

The real question was whether continuity or reality was preferable in the given circumstances. Sometimes one was preferable and sometimes the other. He agreed with the author that the bonus reserve method was an instrument, like the net premium method, with its own uses and limitations. But it was, perhaps, of interest to notice that the American offices, which were essentially wedded to the net premium method and used the contribution method of distribution of surplus, when they had to deal with big changes tackled the problem by means of what they called—in a vivid American phrase—the reservoir of future dividends. That was the bonus reserve method in another form.

On p. 304 the author referred to the market value of assets and the possible use of notional prices. In considering the question, it was necessary to look, not so much at one technique or another, but at the assets. It seemed to him that the freedom which was currently claimed for investment policy would ultimately compel some change in valuation methods. It was difficult to say how that change would come. It might come by a change in the method of valuing assets or a change in the method of valuing liabilities.

In America there had been experiments which were of interest. Instead of assets being valued at market value, a practice was growing up of valuing them

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at a moving average of prices, say over the last five years. That would seem to be a helpful idea for those who set store by continuity.

There might be a change in the valuation of liabilities, but if account had to be taken of investment of a large part of the funds in ordinary shares, he was not sure that any of the suggested methods of valuation, whether bonus reserve or net premium method, would deal with the problem of the wide fluctuations in value and income that were experienced with that type of asset. Freedom in investment policy was the background of the work of the actuary, and it would ultimately, he thought, compel some change.

Mr J. B. H. Pegler observed that the author had headed his paper with an enigmatic quotation from one of the most brilliant younger poetic dramatists. It was not, therefore, surprising that there were in the paper both brilliant and illuminating flashes of insight and provocative statements, which perhaps were not meant to be taken too literally. It was not easy to accept all the author said, but he had shown many aspects of his vast subject in a new and revealing light and, to quote from the same source, he had 'the voice which makes balance-sheets sound like Homer'.

The earlier part of the paper was concerned with the author's theory of immunization, with some of its limitations and with the practical difficulties in giving effect to it. The subject had been very fully discussed in the paper by Haynes and Kirton. He would content himself, therefore, with paying a tribute to the elegance of the author's mathematical demonstration of his theory of immunization and rubbing in, if he might, what appeared to be the most important lesson of his analysis. It could hardly be said too often that the safe course for the investment of most, if not all, life funds was, as Mr Rich had pointed out, to invest long. Any departure from a long-term investment policy was a departure from the normal. Such a departure might be justified; it might be highly desirable. But it was a risk, and a risk which should only be run with the actuary's eyes wide open.

The most controversial part of the paper was the discussion of the relative merits of different methods of valuation. The author discussed valuation as an instrument, a means to an end, the end being either the proof of the company's solvency or the control of the emergence of surplus. Without something approaching the author's great practical experience of valuations, it was not safe to challenge his views as to the most suitable instrument to use, at any rate for the second purpose. Nevertheless, he wondered whether the master craftsman was necessarily the best adviser on the most suitable instrument for the less skilful to use. The author's skill and experience were such that he could do what he wanted, even with the second best.

The speaker's main difficulty with the net premium method was that the net premium reserve was the same for with-profit as for without-profit contracts for the same age and duration. If a net premium valuation at a lower rate of interest than current conditions would dictate were used to hold back surplus, the extent of the holding back must depend on the relative proportions of with- and without-profit business. If the proportions changed, the stability would be disturbed. Perhaps the author met that difficulty by using different rates of interest for the valuation of with- and without-profit business, but he could not remember seeing that mentioned in the paper. It might be that in practice—and he must admit that his own practical experience was small—the point was of little importance.

When it came to the philosophical basis even a novice on the practical side might be bold enough to put forward an opinion. The author regarded valuation as an instrument, and he rightly pointed out that for certain purposes it was a rather crude instrument. It could not, he said, reflect in detail the variety and complexity of events, and therefore, he implied, there was no point in trying to make it do so as nearly as possible. That approach did not seem to him wholly correct. It was necessary also to regard the results of a valuation as a statement of fact or opinion; and if an actuary made a statement of fact or opinion, that statement must be as near the truth as possible. But valuations made on bases which were not as nearly as possible in line with what, in the actuary's opinion, would be the future experience as to interest, mortality and expenses could not be regarded as the best approach to the truth. For that reason he was unhappy at letting expediency, however cogent the argument, play an overriding part in the choice of bases.

The author distinguished most properly between valuation for solvency and valuation for distribution of surplus. Since in practice, as he pointed out, solvency was seldom in doubt, the main concern was the valuation for distribution of surplus. Was it not possible to reconcile the requirements of equity and of truth by making a valuation on bases which accorded most nearly, in the actuary's view, with the truth and controlling the emergence of surplus in the light of the analysis of surplus?

It was a view held by accountants, he believed, that a company's balance-sheet was intended, not to show the real value of the assets, but only to be a historical record of the way in which money had been spent. They, no doubt, contended, on much the same grounds as did the author, that to alter the value of the assets in the light of current conditions would upset the emergence of profit. Such a view was rather unsatisfactory. He knew very well that it was no easy matter to decide on the value of an asset or liability: but the fact that the value was difficult to assess was not a good reason for not trying to assess it. No person responsible for a balance-sheet should be happy to see on it values which though true at some date in the past did not reflect current conditions. He shared the desire of the author's 'intelligent member of the public' to keep in touch with realities.

The author said innocently that he did not wish to exaggerate the difficulties of the bonus reserve valuation, but here, as sometimes elsewhere, his halo of innocence was on at such a rakish angle that it gave him quite an air of iniquity! He had said also that it was odd that those who held the view that such a valuation was scientifically accurate did not 'ask themselves the simple question whether it was in fact right to assume a *uniform* future bonus'. Was it not possible that actuaries who favoured a bonus reserve valuation for internal purposes had asked themselves that question and had decided that, although it was most unlikely that the rate of bonus would remain constant, such an assumption was a convenient way of deciding what proportion of surplus should be held back and what proportion distributed? When the yield of a redeemable security was calculated, it was known that income tax was unlikely to remain unchanged for the rest of the security's life, but a uniform rate of tax was the best guess that could be made and it was almost universally used.

He hoped his remarks had not given a general impression that he was not in sympathy with the author's views. The paper had shown him how little he himself knew about that important subject, and had done more to remove his ignorance than anything he had read since he had studied for the Institute's examinations.

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Mr A. T. Haynes agreed wholeheartedly with the general tenor of the argument set forth in the paper, though he had one or two minor reservations. One of the reservations was that, in relation to immunization, he would regard the author's second-derivative profits as being a product of judgment and not a result of matching. But that was really a philosophical point, and it merely meant that he preferred to take as his standard what the author described as absolute matching (which was only strictly possible for a stationary or decreasing fund), and to regard any other asset distribution adopted in practice as a departure measured from that strict standard.

Far more important was the fact that he would support to the full the main theme of the author that the values of the assets and the liabilities were relative and not absolute. That theme ran right through the paper; it underlay the theory of immunization, and it was inherent in the concept of the estate. The actuary could, on certain assumptions, assess solvency. He could also, on certain principles, assess surplus. But the one thing the actuary could not do *in vacuo* was to place a specific present value on the liabilities. In other words, cash could never be stated to be absolutely equal to future liabilities unless the same amount of cash could also be equated to such future asset proceeds as would meet the future liabilities. What was required was a means of securing a future equation of assets and liabilities, but that was a three-dimensional concept which it was extremely difficult to express in a balance sheet in two-dimensional form.

Perhaps the nearest approach that had been made to expressing the three-dimensional concept in visual form was provided by the author's diagrams on p. 299. The interesting thought which the author produced from those diagrams —that there was such a thing as conditional as opposed to absolute solvency— led to the question whether some forms of conditional solvency could not easily be converted to absolute solvency. For example, where, as in Figs. 2 and 3, the asset and liability value curves might cross owing to a change in the rate of interest, despite the fact that the office was solvent at a given rate of interest, the position could be converted immediately by altering the assets in such a way as to immunize the liabilities. The asset curve would then follow the same form as the liability curve and conditional solvency would be translated into absolute solvency. Similarly, where conditional solvency was due to redemption options, existing assets could be exchanged for matched assets and absolute solvency attained. The one type of conditional solvency which could not readily be converted into absolute solvency was a result of liability options. Such options— guaranteed annuity options, guaranteed surrender values, guarantees of rates under group pension schemes—once granted, remained on the books for years and their potential danger had to be faced. In that connexion he would go further than the author and say that reserves ought to be set up immediately to meet the chance that the options might become onerous and that the actuary's assessment of current surplus should be reduced—in some cases very extensively reduced—by reason of the options having been granted.

Those thoughts raised a most important question: 'What can be done now to protect the position of the fund at future valuations?' The question did not fall strictly within the limits of the subject of valuation but lay rather within the field of new business and investment policy. To his mind, the fundamental answer was to restrict the granting of options to an absolute minimum, and to bear in mind that an immunized asset distribution was the safety standard. Where options were granted, in relation to either assets or liabilities, or where

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judgment was exercised in going long or short in one's asset distribution, ample free reserves should always be held to meet the potential loss. As several speakers had said, the most important free reserve in a life office was created by the with-profit business on its books and that free reserve should be maintained by a sufficient flow of with-profit new business. The maintenance of a proper proportion of with-profit business was an important factor to many offices when large quantities of non-profit business were being transacted and when the 'gearing' of their funds was changing rapidly.

He felt that a policy of complete immunization should be regarded as the standard concept for with-profit business—as for non-profit business—subject, however, to the question whether future bonuses should not be left out of account, the value of future bonus loadings being held, in effect, as a free cash reserve instead of being invested in assets designed to immunize future bonuses. There was more than one approach to the immunization of with-profit business but the important point in practice was to create a clear concept and to understand its effect under varying conditions.

Mr Leslie Brown said that, like a previous speaker, he would start by criticizing the use of the word 'immunization'. Whatever the need for it in the theoretical concept of the paper, it implied a precision in the working out of the theory which could not in fact be justified, owing to the many reservations which were necessary in both theory and practice. He would prefer some such expression as 'minimizing the investment risk'.

The main advance in thought in the paper was the attempt to make investments cover the future net income of the existing business. There were two lines of approach to the problem of the effect of future changes in rates of interest. The first was to consider the way in which the business would ultimately work itself out; the main problem there was the possibility of lower rates of interest from the investment of the future net income of the existing business. It was intriguing to think that, if there were a danger of lower rates of interest, it was possible in theory to meet it by investment before the lower rates happened. The second line of approach was by periodic valuations, and there the main danger was that of higher rates of interest which would cause depreciation. If the assets were balanced, whatever might be the conception of that word, the problem descended to one of presentation. If they were balanced, the protection depended upon the position that both assets and liabilities were similarly sensitive to changes in the rates of interest. The effect on liabilities of any given change in interest rates would be precise, being a matter of calculation—subject, of course, to the question of surrender values and other options already mentioned. But a net premium valuation was not adequately sensitive and, if one faced the possibility of extreme movements greater than the reserve margins or buffers could deal with, it was clearly essential to be prepared to adopt a valuation basis which was adequately sensitive to the movement of interest rates —some form of bonus reserve valuation.

On the assets side, however, the effect on market values of a given change in the rate of interest could not be predicted. Markets were governed by too many complicated forces. There was not one rate of interest. The differences between the money market or short-term rate and the long-term rate were such that they could move in different directions at the same time. The existence of long-term options, the fact that different investors were subject to gross and net rates of interest and many other variations made it impossible to forecast exactly what

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would happen to market prices. Not the least important was the variation in the appreciation of the investment risk inherent in the different classes of industrial investment. Some part of that variation would arise from the factors which caused the change in the rates of interest. Clearly, when interest rates were high, as they were to-day, there were inherent risks arising from the economic causes which had caused that change.

Another way of approaching the same problem of the differing sensitivity of assets and of liabilities to the rate of interest would be to start with the assets and ask what was the rate of interest for the mixed fund. The answer was difficult to achieve. His main point, therefore, was that the theory, valuable though it was, was not precise in practice—he hoped that in saying that he had exploded any thoughts that some might have that the control of investment, following publication of the paper, would be simple and automatic.

He had avoided many other problems and had not touched on the questions whether assets of the right term could be obtained in the market, whether any debtor was good enough to be sure that his credit would last, so that a debenture or preference share could be considered to be really perpetual, and that intriguing question, what was the term of an ordinary share?

Despite his doubts about the precision of the theory, he felt that it made a valuable improvement in the instrument for measuring the position of the life fund by the relationship of assets and liabilities. It was possible to determine in broad measure what might be called a normal position. That would give an idea, a reasonable idea, of the degree of departure from that position and it would provide a background against which to consider investment outlook. Obviously, it was proper to depart from the norm according to the views held as to the future outlook. Obviously also there was room for considerable development of thought on the subject.

Mr S. F. Isaac, in closing the discussion, said that the nature of the territory and the name of the guide were alone sufficient to ensure the success of what was so modestly described as a ramble. Much of the territory was familiar to those who were present, but there could be few who would not have benefited in some way by viewing at least some part of it in a new light or from a new angle.

He would like to join his own tribute to the tributes of earlier speakers to the stimulating original work which the author had done in opening up a new approach to some important territory which until quite recently had been so little explored. He referred, of course, to the work on the matching of assets and liabilities and the possibilities of immunization.

It was important to stress that the author made no extravagant claims for his work in that connexion; indeed, he was at pains to emphasize the limitations of what was, after all, a theoretical examination of the problem. In certain circumstances there was no real solution, and even where there was a real solution it was continuously changing. It was quite clear, however, that much could be done to protect insurance offices, although in most circumstances complete immunization was impracticable and many would say undesirable.

Like Mr Leslie Brown, he hoped that no quotation taken from its context would suggest, particularly to the outside world, that the investment of insurance funds had now been made foolproof by the introduction of some magic formula. The author had demonstrated forcibly how desirable it was that the actuary should play a large part in formulating investment policy, and he did not mind how much publicity that aspect received.

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With regard to the main part of the paper, the author had expressed a preference for the net premium method of valuation because of the light it threw on the anatomy of the business. At the same time he favoured the public presentation of results in the form of a bonus reserve valuation, because of its greater flexibility, and advocated a passive valuation policy because it was conducive to a reasonable degree of equity in the distribution of surplus.

Opinion was still divided, and he supposed it always would be, on the relative merits of the two methods of valuation. Both had their merits and defects. The majority that evening seemed to favour the bonus reserve method of valuation. He must confess that he himself had more than a sneaking regard—he had a warm affection—for the net premium method. But that might only be because, as the author said, it was relatively foolproof!

In normal circumstances, he preferred to value by the net premium method because he felt, with the author, that it threw more light on the anatomy of the business. He preferred to value annually by the net premium method and to publish the results of a net premium valuation, making a gross premium valuation occasionally because of the further light that method could throw on the actuary's problems. He would make a gross premium valuation if there were any major changes in conditions. But, as the author said, the gross premium method did capitalize differences between premium bases and valuation bases in a way that could be misleading and—he thought—sometimes dangerous. It was sensitive to small changes in the rate of interest assumed, and it was also sensitive to small changes in the margins assumed for expenses. That could give rise to difficult problems in the case of funds where the proportion of non-profit business was large. It would be found that a small difference in the provision for future expenses could have a relatively marked effect on the surplus and on the bonus.

He agreed entirely with the author's arguments in favour of a central contingency fund committed neither to the assets nor to the liabilities. It could obviously be of great assistance in avoiding too frequent changes in the values of either. Several speakers had pointed out that it might be required to take care of fluctuations due to a wide variety of causes. It seemed to him that the number of imponderables with which actuaries were concerned in their business tended to grow.

A marked increase in expenses could be a serious matter for a life office, especially if the non-profit business had become relatively large. In recent years the non-profit business of most offices had tended to grow more rapidly than the with-profit business, especially if deferred annuity business were included in a combined life and annuity fund. In that case, the dangers were undoubtedly aggravated. Where with-profit business had become in effect a highly geared equity, it had become extremely vulnerable to increases in the rate of expenses. That was an added argument for building up and maintaining the contingency fund.

The author was right to stress the dangers of options. In recent years many securities had been issued containing options adverse to the investor, but such investments could be avoided, and in his opinion they should be. The institutional investors had to some extent succeeded in making their views known to those people who were concerned with the raising of money and he hoped that as time went on they would be able to make their views still more effectively known.

Options in life assurance and annuity contracts had become an important and inherent feature of the business. It was often necessary to grant such options, but their possible effect on the business should be carefully watched.

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He felt that the author had been a little severe in his criticism of the granting of guaranteed surrender values. After all, however much circumstances might justify reducing surrender values, it was likely that the banks and other lenders would successfully resist any such action. He thought that what was wrong was not the guaranteeing of the surrender values but the granting of over-generous surrender values. It was not unreasonable to guarantee surrender values on a conservative basis, especially if the office reserved the right to postpone payment for a period of, say, six months, which might well tide the office over a crisis.

He agreed with the author and Mr Haynes that some guaranteed settlement options which were granted could be very expensive to companies and might constitute a serious problem, at any rate unless reserves were built up to deal with them. In that connexion, he felt that some of the modern annuity options were difficult to justify.

The President (Mr F. A. A. Menzler, C.B.E.), in proposing a hearty vote of thanks to the author, said that he had on more than one occasion urged the desirability of maintaining a due quota of papers bearing directly on their day-to-day professional work. By every test, Mr Redington's valuable and distinguished paper came into that category.

There had been a full and authoritative discussion to which those who were immediately concerned with the problems of life-office finance had made important contributions. It was therefore quite unnecessary for him, even were he capable of doing so, to attempt to make any critical contribution to the subject of the paper. Nevertheless, he might perhaps permit himself one or two general observations.

He had long felt that as a profession they tended to be too much obsessed with present values. Those all too convenient summarizations swept up everything into a single portmanteau figure; but, as was so often the case with financial-cum-statistical summarizations, that clear view of the wood might cause them to forget the trees or, in other words, the series of financial events in time for which they were called upon to make provision.

With the ever-growing significance in the national economy of savings through life assurance and pension schemes, they would have to pay increasing attention to emerging costs and to the resources necessary to meet them without prejudice, of course, to the fundamental necessity of adhering to the funding principle. At the time of the granting of the Charter, they were described as 'scientific financiers', but it was hardly scientific finance to be chiefly pre-occupied with the liabilities side of the balance-sheet. Only in very recent times did they seem to be attempting to evolve a systematic approach in regard to the investment of the assets comparable in authority with that which they had always attached to their rather meticulous procedure for the assessment of the liabilities they had to meet. It was, he suggested, a sign of the times that within a few months they should have had papers before both the Faculty and the Institute—he was referring to the important paper before the Faculty by Messrs Kirton and Haynes on *The financial structure of a life office*—in which systematic attention had been given to the matching of assets and liabilities in life assurance funds.

As he read Mr Redington's paper, he recalled the time when he was called upon to study those matters. There were no textbooks and no actuarial notes. They had to do the best they could with the voluminous papers and discussions recorded in the *Journal*. He could not remember any serious mention of the

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idea of 'matching', but he had read with pleasure the extremely luminous paper produced by S. G. Warner in 1902 (*J.I.A.* XXXVII, 57) in which the pure doctrine of the net premium method was enunciated with authoritative clarity. It was therefore some consolation to find Mr Redington declaring that 'in stable conditions my personal preference is for a net premium valuation'. He found that reassuring until he re-read the passage and noticed the words 'in stable conditions'. It was apparent that, if he was to begin to keep abreast of those matters, he would still have to keep re-reading Mr Redington's paper.

He had derived much satisfaction from the fact that the last ordinary general meeting of the Institute over which it was his duty to preside should have been the occasion of the delivery of a paper which he was confident would come to be regarded as a landmark in the evolution of professional practice in regard to the administration of the finances of life offices.

The author, for whose abilities they all had such a profound respect, had demonstrated that, contrary to what was sometimes believed, mathematicians could also be good practical actuaries.

Mr F. M. Redington, after thanking the speakers for their encouragement, replied briefly but said that, in view of the many points raised in the discussion, he proposed to submit a written reply for publication in the *Journal.*

Mr J. L. Anderson has sent the following written contribution:

I agree with the author that the net premium valuation and the bonus reserve valuation have their own particular uses and limitations. Generally speaking, I think it is desirable that an office should carry out valuations by both methods, a practice which has been followed in my own office for many years.

The method to be favoured must be largely a matter of personal taste, and I confess to a fondness for the particular type of bonus reserve approach which is described on p. 303, i.e. the method by which one first estimates the average bonus earning power of new with-profit business based on current rates of interest, and then calculates the liability under the existing business reserving for this rate of bonus. The excess of the assets at market values over the resulting liability gives what the author describes as 'the estate'. It is then a comparatively simple task to estimate the future trend of bonus earnings on various assumptions as to the rate of bonus declared for the investigation period which has just ended. The future trend can be shown in the form of a table similar to that given on p. 309. The beauty of the method I have described is that the bonus emerging under the existing business and the bonus earned by the new business remain constant, and variations are confined to two additional factors; first, the interest on the estate which, in the case of an expanding fund, will, of course, provide a steadily reducing addition to the bonus earned, and second, the bonus earned from miscellaneous sources such as surrenders.

It is sometimes desirable to produce similar figures based on a different rate of interest. This will entail an estimate of what the market values of the existing investments would be at this different rate, a task which sounds tedious and complicated but in point of fact can be done with sufficient accuracy quite easily. With two sets of figures showing the trend of bonus earning power on two separate assumptions as to the rate of interest, the actuary is in a good position to decide on the bonus policy which he wishes to recommend.

During the period of low interest rates from which we have recently emerged, the estate calculated on the basis I have just described will have been a positive

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quantity. At a time of high interest rates, however, it is quite possible that it would emerge as a negative quantity and there is nothing inherently absurd about this. It would mean, of course, that the bonus earned by the interest on the estate would also be a negative quantity and the effect on a growing fund would be to make for an increase in the rate of bonus earned as the fund increased—a natural enough conclusion in the circumstances. I should perhaps make it clear that I do not favour the use of a bonus reserve valuation for purposes of the published accounts.

The author refers on p. 308 to the possibility of starting a new fund for future business after a big rise has occurred in the rate of interest. Using the figures on that page, it is clear that if a new series were started, holders of recently effected policies on the old series whose bonus had been immunized at 30*s.* would be tempted to surrender their contracts and effect policies under the new series earning 45*s.* This draws attention to the limitations of the theory of immunization. If we consider separately the fund appropriate to a block of business which has been on the books for a long time and the fund appropriate to a recently effected block of business, it is clear that while the investments appropriate to the former fund may be invested sufficiently 'long' to immunize it, the same is not true of the latter fund. While therefore it may be possible to immunize the fund as a whole, this can only be done by investing the assets appropriate to the old business in sccurities of longer average date than would otherwise be necessary. Is it therefore equitable in the circumstances envisaged on p. 308 to regard the bonus earning power of old and recent business alike as 30*s.* % and would it not be more correct to treat the recent business as having a relatively higher bonus earning power?

In a paper read to the Faculty of Actuaries in 1944 (*T.F.A.* XVII, 137), I attempted to measure the relative bonus earning power of different groups of policies, classified according to age, duration in force, etc., after a big change in interest rates. For this purpose I found it convenient to assume for each group a matched set of assets, neglecting future increases in the corresponding fund—i.e. what Messrs Haynes and Kirton, in their recent paper to the Faculty, called the 'hump'—and so avoided the difficulty of immunizing the fund appropriate to a recently effected block of business. This is a convenient method for measuring relative bonus earning power, but it does not follow that it is equally suitable as a guide to the investment spread which will give the best protection to the fund as a whole against a change in interest rates. In fact it is abundantly clear from Mr Redington's paper, as well as from that of Messrs Haynes and Kirton, that it is essential to take account of the 'hump' in order to obtain this protection. I see no reason, however, why the same basis should necessarily be used when considering equity.

I am a strong advocate of the view that equity is best achieved in general by leaving with-profit premium rates unchanged and allowing the bonus to find its new level by degrees. The business effected just before a rise in interest rates will then participate in the higher rates of bonus to be declared in the future (assuming no further change in interest rates) over a relatively big proportion of its lifetime. On the other hand older business, and especially endowment assurance policies nearing maturity, will tend to go off the books before the higher rates of bonus are attained. Rough justice is thus done to all groups and the results are not inconsistent with the limited immunization which I consider it appropriate to assume in measuring bonus earning power. If interest rates fall substantially, the same arguments would apply; but the rate of bonus would, of

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course, tend to fall, and old business would receive higher average rates of bonus over its future lifetime than would business recently effected.

Mr Redington, in his written reply, says:

The discussion, like the paper, falls naturally into two parts: matching and valuation. Dealing with these two subjects in that order, I am appreciative of those many comments which underlined the practical qualifications to the immunization theory. I was anxious that the necessary over-simplifications contained in the theory should not be overlooked; nevertheless, we cannot, in our anxiety to avoid a mistake, avoid all action. The assets must be invested and the actuary must know, however approximately, what term and spread of assets he is aiming at. The majority of speakers seemed to share my view of the importance of the subject and to agree with the broad lines of my approach.

The main line of criticism was in regard to the treatment of with-profit business. I am glad that this was raised because it is an important aspect of the problem, on which I am conscious of having been all too brief. A substantial part of the liabilities of most life offices is attributable to future bonuses. I think that most actuaries would agree that, while it is permissible to 'take a view' in investing any of the assets, more caution should be exercised in the investment of the assets held against the basic contracts than in the investment of the assets held against future bonuses. Whereas security is the primary consideration for the basic contract, maximum profit is an important consideration for bonuses. So far I agree with my critics. But the aim of the immunization theory is to find the neutral term of the investments on the assumption that no view is taken as to the future rate of interest. I assumed—and it is mainly an assumption rather than a principle—that the office would wish as a broad policy to stabilize the bonus earnings from existing business and I sought the neutral term on that assumption. There is much scope for discussion on this point and I would certainly not be rigid about it; but two important points should be borne in mind: (i) if an office departs substantially from the neutral term which stabilizes the bonus earnings, it may be opening up the possibility of a negative bonus in conditions of extremely low rates of interest; (ii) any future new business will, of course, be completely sensitive to interest changes and, if the bonus on existing business is also made sensitive by investing short, the office may find itself embarrassingly sensitive.

Messrs Bayley, Perks and Ogborn all suggested a paid-up-policy immunization for with-profit business. I am not sure what paid-up-policy was to be immunized, but assuming they meant some guaranteed paid-up-policy, incorporated or implied in the contract, the resulting mean term of the investments would be much shorter than that resulting from the method contained in the paper, and would, I think, be much shorter than they would contemplate as a normal practice. It means, in brief, that the existing business as well as new business would be sensitive to interest changes, so that if the rate of interest falls the bonus potential of the whole business would immediately fall and this could occur in circumstances when options both in the assets and liabilities were becoming a serious embarrassment.

Paid-up-policy matching for with-profit business is probably undesirable as a general practice and is certainly not without theoretical defect. This can be seen readily enough by taking as an example an office adopting a policy of low bonuses with small bonus loadings. If the office has been running steadily on a 4% rate of interest and immunizing on a paid-up-policy basis, a permanent fall in the rate

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of interest to, say, $2\frac{1}{2}\%$ could leave its with-profit business insolvent. In short, at the borderline between with-profit and non-profit business paid-up-policy matching is seen to be a different animal. It does not do what it may seem to do on the surface, namely, immunize the basic contract.

Mr Perks underlined the comments made in the paper about the unacceptability of the logical mathematical conclusion that bonuses on a policy should depend on the rate of interest at issue. I feel, however, that any principle of with-profit matching will, if driven to its final logical conclusions, prove to be unacceptable. For example, the logical conclusion of paid-up-policy matching is to declare rates of bonus which vary with and are substantially affected by the rate of interest ruling at the date of valuation.

From these two logical conclusions it follows that:

(*a*) bonuses dependent upon the rate of interest at issue produce an aggregate surplus which conforms with our practice and tradition of regularity, but lead to a differentiation between policies which is contrary to our practice;

(*b*) bonuses dependent upon the rate of interest at valuation lead to instability in aggregate surplus which is contrary to our tradition, but lead to more uniform subdivision between policies.

Mr Haynes pointed out another aspect of the problem: namely that paid-up-policy matching can entail the encroachment on future bonus loadings. I agree with this comment and there is no need to amplify, but it gives me the opportunity to say that if we plunge deeper into the with-profit question we may conclude that a reasonable neutral principle would be to immunize the future cash surplus rather than, as was done in the paper, to immunize the future rate of reversionary bonus resulting from that cash surplus. This leads to a slightly shorter investment policy, but by no means so short as paid-up-policy matching. Perhaps I can summarize my own views by saying that for with-profit business there is some flexibility at the edge of the problem but that it is not possible to depart far from the method adopted in the paper without at least jeopardizing the buffer which with-profit business should provide and at the worst rendering the whole business insolvent. On the whole, for with-profit business I prefer to think that it is legitimate to depart from my conception of neutrality, rather than to think that neutrality lies far away.

I have devoted some time to this with-profit question because it has been the main line of criticism, and is one of the more vulnerable points. It leads me to a wider conception of the whole problem. In the paper I discussed what conditions the invested assets must satisfy to immunize the business against changes in the rate of interest. The problem can be expressed differently. Whenever an office accepts a new contract or makes an investment it affects the balance of the business as regards its sensitivity to interest changes. What is that effect? Subject always to the theoretical simplifications, the Taylor expansion in the paper gives the answer to that question in what I thought would be an uncontroversial manner. There is another and more controversial question as to what is the effect desired.

Comments on the question of valuation were mainly in amplification. More extensive criticism of those sections of the paper dealing with the net premium and bonus reserve valuation came from Messrs Bayley and Ogborn. I think that the difference between us is less a question of fundamental opinion than of the expression of that opinion. I have much sympathy with the view that, as Mr Bayley had expressed it, the direct expression given to the two variables, present and future rates of bonus, under a bonus reserve valuation gives

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greater freedom of manoeuvre. I feel that on the whole, and taking a long period of history, I would find it easier to maintain a satisfactory presentation of results during a long sequence of varied events through the medium of a bonus reserve valuation than that of a net premium valuation. Net premium valuations, however, make it easier to determine what is the right action to take, even if that right action has to be translated into a bonus reserve valuation for the purposes of presentation.

Whatever one does about investments or premiums or options, there is some rate of interest, be it 0% or 10%, for valuing the liabilities which equates them with the market value of the assets. That thought is both illuminating and sobering. It shows how much of the problem lies with the actuary's judgment and indeed with his conscience. It shows how essential—professionally vital—it is that the actuary should be acquainted with events on the investment side of the office. It adds force to the President's comment that 'present value' methods can obscure reality. It also shows that the problem finally lies in the hands of the individual actuary and I am happy to feel that in this country those hands are, and have for many years been, very safe.

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