THE INSTITUTE OF ACTUARIES

THE RATE OF INTEREST WHICH SHOULD BE EMPLOYED IN THE VALUATION OF A PENSION FUND AND THE VALUES WHICH SHOULD BE PLACED ON EXISTING INVESTMENTS

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INTRODUCTION TO THE PROBLEM

1. AFTER the discussion on Sir George (then Mr) Epps's paper on 'Superannuation Funds', the President in proposing a vote of thanks said: 'It is of the highest importance that we should exchange views with one another and try to come to a common opinion on matters of practice so that the world at large will bring against us no charge of inconsistency in the advice which we respectively give to those who consult us.' In 1921, when Epps's paper was presented, actuaries were faced with the position that current yields on new investments were very much higher than had been anticipated in earlier valuations, while most of the older funds had suffered very heavy depreciation in values of existing investments. Today the circumstances are reversed. The rate of interest which can be earned on new investments is very low, but there has been substantial appreciation in values of existing investments. It is in the hope of provoking an exchange of views on the best method of dealing with this present situation that this paper has been written.

2. The guidance which is given on this matter to the student who is qualifying to value pension funds is of a purely general nature and is likely to give him only very limited assistance in making a decision in regard to a particular fund. In D. A. Porteous's book *Pension and Widows' and Orphans' Funds*, there is only one paragraph on the subject of interest which reads as follows: '...the rate of interest...has to be decided with reference to (i) the yield of the existing fund, (ii) the relative size of the annual sums which will have to be invested in the future, and (iii) the probable yield on these future investments. It must also not be overlooked that the rate adopted involves the assumptions that it will be realized over a very long period in the future and in these circumstances, some margin must be retained in relation to the rate actually yielded by the existing fund.' At the present time, the margin between the 'yield of the existing fund' at balance-sheet valuation and the 'probable yield on future investments' is frequently very large, and this general advice needs considerable amplification.

3. It is not proposed to discuss at length the considerations which will guide the actuary in arriving at a decision in regard to the probable average rate of interest which a given fund is likely to earn on future investments. The existing portfolio will supply an indication of the past investment policy, account will have to be taken of the investment powers vested in the trustees, and then such allowance as may be deemed necessary must be made for the probability that money is likely to become cheaper or dearer in the future. Throughout this paper, however, it is assumed that this admittedly difficult

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decision has been made and that the problem to be solved is how best (*a*) to calculate the consequent deficiency (or surplus) and (*b*) to present the results of the valuation so that they shall be understood by trustees and employers. In the present circumstances it is no longer possible to pass directly from 'the probable yield on future investments' to a suitable rate of interest for a valuation, in which credit is to be taken for existing investments at balance-sheet values, merely by a general study of (1) the yield of the existing fund and (2) the relative size of the annual sums which have to be invested in the future, nor is there any need to attempt to do so. A much simpler plan is to value assets (including existing investments) and liabilities at the rate of interest which it is anticipated can be earned on future investments.

4. It is sometimes stated that so many 'imponderables' have to be taken into account when making a pension fund valuation that the result can be regarded as nothing but an approximate indication of the financial position of the fund at a given moment. A general statement such as this, although undoubtedly true, does not absolve the actuary from a responsibility to set out clearly the assumptions which he decides to make, and then to produce an answer such that there will be no further surpluses or deficiencies at future valuations if (*a*) the future experience of the fund follows exactly these assumptions (however improbable this may be) and (*b*) action has been taken to make good any deficiency shown at the current valuation. In order to ensure that such an answer does emerge, the actuary must take into account the true value of the existing portfolio of investments and the probable expected strain (if any) through the admission of future entrants.

ALTERNATIVE METHODS OF APPROACH

5. The straightforward method of procedure is to value at the rate of interest which it has been decided to assume is likely to be earned on future investments and make direct allowance for:

(i) future interest from existing investments in excess of the valuation rate,
(ii) anticipated profits or losses on redemption of existing investments,
(iii) expected strain (if any) through admission of new entrants.

Alternatively, it is possible to make a valuation which does not take direct account of appreciation or expected strain (if any) through admission of future entrants and to produce the same answer by employing a rate of interest such that indirect provision is made for these features. It may, however, be much more difficult to explain the deficiency to the trustees or employers.

6. If the straightforward method be adopted, the trustee or employer will receive a report which will say in effect: 'It is considered improbable that a rate of interest higher than $i\%$ can be earned on future investments of the fund. All existing assets and liabilities have accordingly been valued at this rate of interest and there is a deficiency of £x. Further deficiencies will arise in respect of each new entrant in the future unless contribution rates are recalculated at $i\%$.' When the indirect method is used, the report will say in effect: 'It is considered improbable that a rate of interest higher than $i\%$ can be earned on future investments of the fund. Bearing this in mind and taking into consideration that rate $j\%$ is at present being earned on the existing investments, the maximum average rate of interest which it can be hoped to maintain in the future is $k\%$, and this rate has been used for valuing the future

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contributions and benefits of existing members and pensioners. The resulting net liability for existing members exceeds the assets as shown in the balance-sheet by £x. It must be clearly understood that no part of the appreciation in the values of existing investments may be set off against this deficiency, as indirect credit has been taken for the enhanced values when it was decided to value at k%. Further deficiencies will arise in respect of each new entrant in the future unless contribution rates are recalculated at i% (or k%?).'

7. The method of presentation of the results is, of course, a matter of personal preference. The point which it is desired to emphasize is that, before making a valuation, the actuary should set out his valuation assumptions clearly and in full, and that once this has been done there can be only one correct answer. Later in this paper an indication is given of the magnitude of the error which is liable to be introduced if an arbitrary decision in regard to a valuation rate of interest be made.

8. The surplus or deficiency of a hypothetical fund which has reached a truly stationary condition can be arrived at quite independently of the value of investments, provided that the rate of interest, say i, at which future investments can be made, is correctly forecast. For such a fund, both yearly outgo for benefits and expenses (if any) and yearly income from contributions will be constant and the difference between them equal to I, say, will be the annual income required in perpetuity from interest earnings. The equivalent annual income in perpetuity from existing investments can readily be calculated by arranging them according to year of redemption and tabulating the amounts to be reinvested in each future year, and the annual interest payable until redemption. If in a particular year of redemption R_t will be repaid and the income for the t years to redemption date is I_t, then the equivalent annual income in perpetuity from this batch of investments is

$$i\{R_t+(I_t-iR_t)\,\bar{a}_{\overline{t}|}\}=I'_t, \text{ say,}$$

where $\bar{a}_{\overline{t}|}$ is calculated at rate i, and the total equivalent annual income in perpetuity from all existing investments is $\sum_{t=1}^{\infty} I'_t = I'$, say. The deficiency will then be equal to $(I-I')/i$, and if a result which differs materially be produced there must be something wrong with the basis or some feature must have been ignored.

9. For a fund which has not reached a stationary condition, it can be demonstrated that the only way in which the problem can be approached scientifically is by first making a valuation of all assets and liabilities at the rate of interest which it has been decided is likely to be earned on future investments. Once this has been done, a rate of interest to be used for the official valuation can be selected which will produce the same answer without taking direct account of appreciation or expected strain (if any) through admission of new entrants.

10. Basic principles can most easily be understood by making tests on actual funds. This is not possible in a theoretical paper, but certain model funds have been constructed which will serve for the purpose of practical demonstration.

PARTICULARS OF MODEL FUNDS

11. Brief particulars of four model funds which will be considered are given below:

Fund	Date when fund first established	Rate of interest used for calculation of rates of contribution (%)
A	1 January 1907	4
B	1 January 1917	4
C	1 January 1927	$4\frac{1}{2}$
D	1 January 1937	$3\frac{1}{2}$

The experience of all funds as regards entries, withdrawals, mortality, retirement and salaries follows exactly the service table shown in Appendix A. The l_x column in the service table, therefore, represents the initial and permanent membership which is maintained by 1000 entries on 1 January in each year. After the fund is established, 670 members retire in each year at exact age 60 and, after the fund has been established 40 years, the l^r_x column in the service table represents the permanent membership on pension.

12. Each of the funds provides the following benefits:

On withdrawal: Return of members' contributions without interest.

On death: A lump sum payment equal to 10% of total past salaries.

On retirement at age 60*:* Annual pension of two-thirds of average salary during last five years of service.

13. The rates of contribution are:

By members: 5% of salary.

By employers: Balance of total percentage contribution of salary certified as required for a new entrant aged 20 at date when fund was established, i.e. 4·681% of salary to Funds A and B, 4·029% of salary to Fund C and 6·962% of salary to Fund D. (See Appendix B.)

14. For each of the funds it has been assumed that the initial deficiency for existing employees was calculated at the rate of interest used for obtaining contribution rates and that the employers agreed to deal with it by making 40 equal annual payments to the fund. The employers are also assumed to take responsibility for all management expenses. The assumptions made as regards benefits and contributions have been selected purely from the point of view of expediency and to reduce the number of calculations. Withdrawals have been restricted to the first 10 years of service and the same death benefit has been made payable for contributory and non-contributory service, so that each fund reaches a stationary condition as regards active members in 10 years from its inception. Ill-health retirements have not been introduced, in order that the funds shall be completely stationary as regards payments for benefits and receipts from contributions 40 years after the dates when they are assumed to have been established.

15. Monetary functions have been compiled at three rates of interest, viz. $2\frac{3}{4}$%, $3\frac{1}{2}$% and $4\frac{1}{4}$%, and thence rates of contribution and initial net liabilities have been calculated and valuations have been made at the end of 10, 20, 30 and 40 years at each rate of interest. Corresponding rates of contribution and net liabilities at other rates of interest have been obtained by second-difference interpolation and the results are shown in Appendix B.

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16. In Appendix C an assumed position is shown for the existing investments of each of the four funds at 1 January 1947 and Funds B, C, and D have each been traced to the date 40 years after they were first established, assuming that all investments after 1 January 1947 will be in irredeemable securities purchased at prices to yield $2\frac{3}{4}\%$ in perpetuity, and that rates of contribution for present members and future entrants will remain unaltered.

For each of the funds it has been assumed that a substantial proportion of the existing investments is in irredeemable or long-dated securities, and the cost prices shown in column (1) have been fixed having regard to the prices at the dates when money would have become available. In column (4) the values of the existing investments to yield $2\frac{3}{4}\%$ in perpetuity are given. Details of the assumed investments of the four funds have been omitted deliberately, as these details have no significance for the present purpose. The figures supplied are reasonably typical and will do as well as any others to demonstrate the effect of the various possible methods of valuation when current yields have fallen and, in consequence, existing investments have appreciated in value. Different assumptions in regard to the investment policy would produce different answers, but the principles would be the same and the conclusions would vary only in degree. For the same reason it is not proposed to discuss the assumption that future investments will yield a rate of interest of $2\frac{3}{4}\%$. This is merely the rate of interest which is assumed to have been adopted by the actuary as likely to be earned by these particular funds on future investments.

POSITION OF FUND A WHICH IS IN A STATIONARY CONDITION

17. The deficiency for Fund A at 1 January 1947, calculated as indicated in paragraph 8, is:

	£000's
Annual outgo:	
Pensions	3,305·2
Death benefits	165·4
Withdrawal benefits	5·6
	3,476·2
Annual income from contributions	1,140·6
Difference = annual interest income required	2,335·6
Annual interest in perpetuity equivalent to expected interest on fund	2,090·0
Annual interest deficiency	245·6
Present value of deficiency at $2\frac{3}{4}\%$	8,931

Unless contribution rates for new entrants are to be amended, this deficiency must be produced in the valuation, or the actuary will have departed from his initial assumption that future investments will earn $2\frac{3}{4}\%$.

18. In the following table the financial position of Fund A is analysed using valuation rates of interest of 4%, $3\frac{1}{2}\%$ and $2\frac{3}{4}\%$ respectively.

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Future investment rate of interest $=i$		$2\frac{3}{4}\%$	$2\frac{3}{4}\%$	$2\frac{3}{4}\%$
Valuation rate of interest $=j$		4%	$3\frac{1}{4}\%$	$2\frac{3}{4}\%$
		£000's	£000's	£000's
(i)	Net liability for existing members and pensioners	58,389	67,421	74,704
(ii)	Present value of strain through admission of future entrants (1000 p.a. each aged 20) at current contribution rate	—	4,443	10,227
(iii) $=$ (i) $+$ (ii)		58,389	71,864	84,931
(iv)	Fund at cost	60,000	60,000	60,000
(v)	Reserve required if rate j to be maintained on existing fund in perpetuity	7,750	—	—
(vi)	Appreciation for which credit can be taken if fund revalued to earn rate j in perpetuity	—	4,308	16,000
(vii)	Adjusted Fund $=$ (iv) $-$ (v) $+$ (vi)	52,250	64,308	76,000
(viii)	$=$ (iii) $-$ (vii)	6,139	7,556	8,931
(ix)	Annual interest deficiency (viii) $\times j$	245·6	245·6	245·6
(x)	Present value of deficiency at rate i	8,931	8,931	8,931

19. The correct annual interest deficiency is produced in all three cases, but only when $2\frac{3}{4}\%$ is used is the correct deficiency shown in line (viii). When a rate is used other than the rate at which future investments can be made, it is necessary first to calculate the annual deficiency in future interest income and then to recapitalize at the rate at which money would be invested if the deficiency were paid up.

20. Another point in favour of adopting the future investment rate of interest as the valuation rate is that by this method the true position is shown as regards existing members and pensioners and future entrants. Only in the case of the $2\frac{3}{4}\%$ valuation is it clear that if Fund A were to be closed to future entrants, it should run off quite successfully without becoming insolvent.

21. It should be noted that in all three cases the adjusted fund for which credit has been taken was determined by valuing existing securities, so that a yield at the valuation rate can be expected in perpetuity, allowing in the case of redemptions for reinvestment at $2\frac{3}{4}\%$. If existing securities are valued on this basis, the correct annual interest deficiency can be obtained in respect of the stationary fund whatever rate of interest be used, provided that both present and future members be considered. The lower the rate of interest employed the greater will be the annual interest deficiency in respect of future entrants. If a rate above 4% be used, credit will have to be taken for anticipated surplus in respect of future entrants to produce the result.

22. If the actuary wishes

(*a*) to take credit for the existing fund at the value shown in the balance sheet (i.e. securities at cost prices without any adjustment for appreciation),

(*b*) to make no specific reserve for strain (if any) through admission of future entrants, and

(*c*) to choose a valuation rate of interest such that the true position of Fund A, if future investments will yield only $2\frac{3}{4}\%$, will be disclosed,

he must first decide what recommendation he intends to make in regard to future rates of contribution and then determine the valuation rate of interest. Two simple demonstrations will serve to indicate how this may be done:

(1) If it be decided to continue to admit future entrants at the present contribution rates then the deficiency shown at this valuation should be

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£8,931,000 and, since the fund is to be valued at £60,000,000, the net liability must be £68,931,000. By interpolation, using net liabilities obtained from Appendix B, it is found that £68,931,000 is the net liability when the valuation rate of interest is 3·1417%.

(2) If it be decided to recommend that rates of contribution for future entrants should be recalculated on a $2\frac{3}{4}$% basis, then a surplus of £1,296,000 should be shown at this valuation. As in the previous case, the fund is to be valued at £60,000,000, so that the net liability must be shown as £58,704,000 which, proceeding as before, is found to be the result obtained by valuing at 3·9719%.

Without the preliminary investigations, it is difficult to see how the correct rate could have been ascertained in either case, and it might be difficult to explain why such an unusual rate of interest had been used.

23. In practice, an actuary, who intended to recommend a revision of future rates of contribution and to take credit for the existing fund at the value shown in the balance-sheet, might endeavour to choose a rate of interest which would show the correct position if used for the valuation and also for the calculation of the revised contribution rates, and thus avoid negative values at subsequent valuations. If this were the intention, the rate of interest to be used could be found approximately by dividing the equivalent annual interest in perpetuity shown in Appendix C, i.e. £2,090,000, by £60,000,000 so that the fund to earn the valuation rate of interest in perpetuity would be equal to the balance-sheet value of the fund. The rate of interest thus obtained would be 3·4833% and, by interpolating for the net liability, it is found that, using this rate, a deficiency of £4,407,000 in respect of existing members and pensioners results. The anticipated deficiency in respect of future entrants would be dealt with by recommending a revision of contribution rates for new entrants from 9·681% to 11·201%. This method also is open to the objection that the deficiency if paid up would have to be invested to earn 3·4833% in perpetuity, and if in fact only $2\frac{3}{4}$% can be obtained further deficiencies would arise through inadequate interest receipts. The true deficiency, if contribution rates for future entrants are revised from 9·681% to 11·201% and future investments are made at $2\frac{3}{4}$%, is found as follows:

	£000's
Net liability at $2\frac{3}{4}$% for existing members and pensioners	74,704
Invested funds at $2\frac{3}{4}$% (see Appendix C, col. (4))	76,000
Surplus in respect of existing members and pensioners	1,296
Value of deficiency in contributions of future entrants (1000 per annum each aged 20) when contribution rate is revised to 11·201%	6,411
Net deficiency for present and future members	5,115

24. By interpolation it can be established that 3·4498% is the rate of interest which should be employed if existing investments are to be brought in at balance-sheet values and rates of contribution for future entrants are to be recalculated at the valuation rate of interest. The deficiency in respect of existing members and pensioners using a rate of interest of 3·4498% is £4,840,000 and the rate of contribution to be paid by future entrants is 11·311%. These conclusions can be readily checked by the straightforward method advocated in this paper, whereby the full value of the existing fund is

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regarded as one of the assets of the existing members and a separate reserve is made for the inadequate contributions of future entrants:

	£000's
Surplus in respect of existing members and pensioners	1,296
Value of deficiency in contributions of future entrants (1000 per annum each aged 20) if rate of contribution is 11·311%	6,136
Net deficiency for present and future members	4,840

25. It is unlikely that an actuary who intended to bring in the balance-sheet value of the fund in hand and to recalculate contributions at the valuation rate of interest would employ a rate of interest of 3·4498%. It may be instructive to show by how much the deficiency would be overestimated if he decided to use 3¼%. The valuation balance-sheet would then show:

	£000's
Net liability at 3¼% for existing members and pensioners	67,421
Fund in hand as per balance-sheet	60,000
Deficiency	7,421

The true deficiency, if contribution rates for future entrants are revised to 11·962% (the rate required at 3¼%) and future investments are made at 2¾%, is as follows:

	£000's
Surplus in respect of existing members and pensioners	1,296
Value of deficiency in contributions of future entrants (1000 per annum each aged 20) when contribution rate revised to 11·962%	4,501
Net deficiency for present and future members	3,205

26. The vast difference between the true position and that disclosed by a valuation at 3¼% serves to illustrate how inelastic is the position if the actuary decides to bring in existing securities at balance-sheet values and to ignore appreciation. In this particular example, when 3½% is used the deficiency is underestimated, and when 3¼% is used the deficiency is inflated from £3,205,000 to £7,421,000. The sceptical trustee or employer who, faced with an actuary's report showing a deficiency of £7,421,000 and recommending that contribution rates for future entrants should be revised from 9·681% to 11·962%, decided to take no action in regard to the deficiency on the ground that appreciation had not been taken into account, would, in this case, be partially justified.

27. It is submitted that the foregoing investigations indicate that, for a fund which is approaching or has reached a stationary condition, the position will be much clearer if the valuation be made at the rate at which it is expected that future investments should be available and existing investments be valued to earn this rate of interest; trustees or employers should more readily understand the position and less might be heard of the phrase 'only an actuarial deficiency'. There is the further advantage that the revision of contribution rates for new entrants, using the valuation rate of interest, makes them self-supporting and that an influx of new entrants in excess of expectations will not create a further deficiency.

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POSITION OF FUNDS B, C AND D WHICH HAVE NOT REACHED A STATIONARY CONDITION

28. Up to this stage consideration has been given to the position of Fund A only, which is in a stationary condition as regards income from contributions and outgo for benefits. It is now proposed to examine the position of Funds B, C and D. The true deficiencies in respect of these funds can be obtained by the same method as was used in paragraph 17 to obtain the true deficiency for Fund A. In each case the interest deficiency at the date when the fund will have been established for 40 years can be capitalized and then discounted to the present date at $2\frac{3}{4}\%$.

	Fund B	Fund C	Fund D
Date when fund will have been established for 40 years	1. 1. 57	1. 1. 67	1. 1. 77
	£000's	£000's	£000's
Annual outgo at the above date	3,476·2	3,476·2	3,476·2
Annual income from contributions at the above date	1,140·6	1,063·8	1,409·4
Difference = annual interest income required at the above date	2,335·6	2,412·4	2,066·8
Annual interest in perpetuity equivalent to expected interest on fund at the above date	1,965·3	1,716·0	1,705·0
Annual interest deficiency	370·3	696·4	361·8
Capitalized value of deficiency at $2\frac{3}{4}\%$ at above date	13,465	25,324	13,156
Present value at 1 Jan. 1947 of expected deficiency at above date	10,266	14,720	5,830

29. As in the case of Fund A, these results can be obtained exactly by valuing liabilities and assets at $2\frac{3}{4}\%$ and considering both present members and future entrants.

	Fund B £000's	Fund C £000's	Fund D £000's
Net liability for existing members and pensioners	74,665	74,872	63,712
Invested funds valued at $2\frac{3}{4}\%$ (see Appendix C, col. (4))	60,958	46,611	28,217
Value at $2\frac{3}{4}\%$ of outstanding instalments in respect of initial deficiency	13,668	25,403	34,166
Deficiency in respect of existing members and pensioners	39	2,858	1,329
Value of deficiency in contributions for future entrants (1000 per annum each aged 20)	10,227	11,862	4,501
Net deficiency for present and future members	10,266	14,720	5,830

30. If the fund in hand is to be brought in at the value at which it is shown in the balance-sheet, i.e. at cost price, the rate of interest which can be used to produce the true result will again depend on the action which it is intended to recommend should be taken in regard to contribution rates for future entrants. For each of the funds, the appropriate rate of interest has been found by interpolation

(*a*) if future entrants are to be admitted at present rates of contribution,

(*b*) if rates of contribution for future entrants are to be recalculated at $2\frac{3}{4}\%$ and amended to 13·755%,

(*c*) if rates of contribution for future entrants are to be recalculated at the valuation rate of interest.

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The following are the results produced:

	Fund B			Fund C			Fund D		
	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)
Valuation rate of interest %	3·0069	3·8504	3·3715	2·6897	3·7035	3·2642	2·7064	3·1638	2·9573
Rate of interest % employed for calculation of rates of contribution for future entrants	4·0000	2·7500	3·3715	4·2500	2·7500	3·2642	3·2500	2·7500	2·9573
Rate of contribution % of salary for future entrants	9·681	13·755	11·566	9·029	13·755	11·916	11·962	13·755	12·989
	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
Net liability for existing members and pensioners	70,771	60,028	65,811	75,870	61,831	67,356	64,391	57,963	60,734
Value of outstanding instalments in respect of initial deficiency	13,505	12,989	13,278	25,550	23,373	24,281	34,361	32,434	33,282
Invested funds as per balance-sheet	47,000	47,000	47,000	35,600	35,600	35,600	24,200	24,200	24,200
Deficiency in respect of existing members and pensioners	10,266	39	5,533	14,720	2,858	7,475	5,830	1,329	3,252

31. These results can be verified against the deficiencies if liabilities and assets are valued at $2\frac{3}{4}\%$ and consideration is given to both existing members and future entrants.

	Fund B			Fund C			Fund D		
	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)
Rate of contribution % of salary for future entrants	9·681	13·755	11·566	9·029	13·755	11·916	11·962	13·755	12·989
	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
Deficiency in respect of existing members and pensioners	39	39	39	2,858	2,858	2,858	1,329	1,329	1,329
Expected deficiency in respect of future entrants	10,227	—	5,494	11,862	—	4,617	4,501	—	1,923
Total deficiency in respect of existing members and pensioners and future entrants	10,266	39	5,533	14,720	2,858	7,475	5,830	1,329	3,252

CONCLUSIONS

32. The conclusions which can be reached from the foregoing investigations may be briefly summarized as follows:

(i) Having set out clearly the assumptions which are to be made in regard to future experience, including the average rate of interest i on future investments, the deficiency can be determined by valuing all assets and liabilities at this rate.

(ii) The deficiency can be divided into (*a*) deficiency in respect of existing members, (*b*) deficiency in respect of future entrants if accepted at current contribution rates.

(iii) (ii)(*b*) can be eliminated by the introduction of a new scale of contributions calculated at rate i, since future entrants will then be self-supporting.

(iv) A revision of contribution rates so as to make future entrants self-supporting is desirable in any event, since otherwise an influx of new members in excess of expectations will produce further deficiencies at future valuations which could by this means be avoided.

(v) If it be decided not to increase rates of contribution for new entrants so that they become self-supporting, indirect allowance can be made for anticipated future strain. By including assets at a value materially less than their real worth at the date of the valuation and adjusting the net liability in respect of existing members by selecting a suitable valuation rate of interest, the deficiency in respect of existing members is inflated to include the expected deficiency in respect of future entrants.

(vi) The suitable rate of interest to use in a valuation, if assets are to be included at a value materially less than their real worth at the date of the valuation, can be determined only by working backwards from the answer obtained when all assets and liabilities are valued at rate i.

(vii) The method of presentation of results is a matter of personal preference but there is only one answer which truly reflects the financial position of the fund in the light of the actuary's initial assumptions.

(viii) A decision to value at a rate of interest fixed arbitrarily may result in the production of a deficiency which differs materially from the true deficiency if future experience follows exactly the assumptions which the actuary decided to make at the outset.

(ix) A valuation of all assets and liabilities at rate i is to be preferred to a valuation which produces the same result by an indirect method, because (*a*) it is likely to be more clearly understood by employers and trustees, (*b*) a very considerable amount of preliminary work is necessary in order to determine scientifically the appropriate valuation rate of interest which should be employed when credit is taken for fixed values of existing securities, (*c*) a valuation rate of interest greater than i will introduce inevitable problems in regard to negative values at subsequent valuations if contribution rates for future entrants are recalculated at rate i.

VALUATION OF EXISTING ASSETS

33. There are, of course, practical difficulties to be faced if the assets are to be valued at the valuation rate of interest. The yield on securities of similar status generally varies with the term and in the calculations allowance must be made for their respective redemption dates. As indicated earlier, the average

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future rate of interest will have been determined by considering the average rate at which investment could be made at the present time, taking into account the range of investments authorized for the particular fund and the proportion likely to be invested in the various classes; adjustments will have been made to allow for any interest margin that it is desired to incorporate and for the view taken of the probable trend of interest rates over a long period. The resulting rate adopted as the valuation rate may therefore be either higher or lower than the average redemption yield on the existing assets taken at market prices; the total value obtained for the assets is likely to be correspondingly lower or higher than the total market value. In the former case no real problem would arise, providing all the securities are marketable and no assets of doubtful security are included. The fact that individual securities may have been valued at prices above the market quotation could be ignored, since the total value adopted for the fund would be less than the total market value. In the latter case it would in practice be difficult to adopt any value in excess of the total market value, and the balance would therefore be retained as a margin which would fall into surplus in the future as the excess interest is received.

34. Where the investments authorized cover a wide range, e.g. including ordinary shares and other securities where a higher yield is coupled with a greater degree of risk, some part of the yield should be treated as a risk premium, and a margin allowed in determining the valuation rate of interest to be adopted. Accordingly, it would not be sound to take such securities at any value in excess of market value, whatever the total market value of the assets may be. It would also be unwise to take any value in excess of the individual market values for

(*a*) securities which are actually in default,

(*b*) securities quoted at a price which indicates some doubt as to the future, or

(*c*) securities in industries threatened by nationalization.

Securities which have no market quotation present another problem. In some such cases it might be possible to obtain an estimated value from a stockbroker but each would have to be considered on its merits. Where the proportion of such assets to the total is small, the safe course would be to include them at cost price, or at the value to earn the valuation rate of interest, whichever is the less.

35. There are many vital problems arising from the economic results of a fall in interest yields which affect the future of pension funds, the most notable of which is the probable effect on future salary scales. These problems have been ignored deliberately, as it is feared that they would divert attention and that the object of this paper would be destroyed. The question for discussion is: 'Having decided to assume that the fund to be valued may be expected to earn a rate of interest of $i\%$ on future investments, how should the actuary determine the rate of interest to be employed for the current valuation and what values should he place on existing investments?'

36. In conclusion, grateful thanks are recorded to Mr R. J. W. Crabbe for his helpful encouragement and criticism and to Mr G. W. Haslam, who has undertaken the numerical calculations and without whose assistance this paper might never have been completed.

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APPENDIX A

Service Table

Age x	l_x	d_x	w_x	s_x	Age x	l^r_x	d^r_x
20	1000	1	35	160	60	670	14
21	964	1	30	180	61	656	15
22	933	1	25	200	62	641	16
23	907	1	21	215	63	625	17
24	885	2	17	230	64	608	18
25	866	2	13	245	65	590	19
26	851	2	10	260	66	571	20
27	839	2	6	275	67	551	21
28	831	2	3	290	68	530	22
29	826	2	1	305	69	508	23
30	823	2		320	70	485	24
31	821	2		335	71	461	26
32	819	2		350	72	435	27
33	817	2		360	73	408	28
34	815	2		370	74	380	29
35	813	2		380	75	351	30
36	811	3		390	76	321	30
37	808	3		400	77	291	29
38	805	3		405	78	262	28
39	802	3		410	79	234	27
40	799	3		415	80	207	26
41	796	3		420	81	181	25
42	793	3		425	82	156	23
43	790	4		429	83	133	21
44	786	4		432	84	112	19
45	782	4		435	85	93	17
46	778	4		438	86	76	15
47	774	5		440	87	61	13
48	769	5		442	88	48	11
49	764	5		444	89	37	9
50	759	6		446	90	28	7
51	753	6		448	91	21	6
52	747	7		450	92	15	5
53	740	7		452	93	10	3
54	733	8		454	94	7	2
55	725	9		456	95	5	1
56	716	10		458	96	4	1
57	706	11		460	97	3	1
58	695	12		462	98	2	1
59	683	13	r_{60}	464	99	1	1
60	—	—	670	—	100	—	—

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APPENDIX B

Rate of interest i (%) ...	2½	2¾	3	3¼	3½	3¾	4	4¼	4½
(1) Rate of contribution % of salary	14·733	13·755	12·831	11·962	11·147	10·387	9·681	9·029	8·432
	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
(2) Initial net liability	44,044	42,447	40,950	39,554	38,225	36,950	35,706	34,496	33,292
(3) Corresponding annual payment (in advance) for 40 years	1,712	1,716	1,720	1,725	1,729	1,733	1,735	1,735	1,731
(4) Value of future benefits to existing members and pensioners after:									
10 years	89,481	84,933	80,733	76,876	73,294	69,958	66,831	63,916	61,164
20 years	95,527	90,889	86,606	82,674	79,021	75,617	72,419	69,432	66,601
30 years	96,485	91,839	87,548	83,608	79,948	76,537	73,332	70,333	67,493
40 years	96,525	91,878	87,587	83,647	79,987	76,575	73,370	70,371	67,530
(5) Value of contributions of 1 % of future salaries of existing members	1,829	1,774	1,723	1,676	1,632	1,589	1,547	1,507	1,466
(6) (5) × 13·755	25,158	24,401	23,700	23,053	22,448	21,857	21,279	20,729	20,165
(7) (5) × 11·962	21,876	21,221	20,612	20,049	19,517	19,008	18,511	18,027	17,540
(8) (5) × 9·681	17,705	17,174	16,681	16,226	15,796	15,383	14,981	14,589	14,196
(9) (5) × 9·029	16,514	16,017	15,557	15,133	14,735	14,347	13,968	13,607	13,237
(10) Value of contributions of 1 % of future salaries of 1000 entrants at age 20	72	69	66	63	61	58	56	54	52
(11) (10) ÷ i	2,885	2,510	2,202	1,948	1,735	1,554	1,399	1,265	1,148
(12) (11) × {(1) − 11·962}	7,994	4,501	1,913	—	−1,414	−2,448	−3,191	−3,711	−4,053
(13) (11) × {(1) − 9·681}	14,573	10,227	6,939	4,443	2,543	1,097	—	− 825	−1,434
(14) (11) × {(1) − 9·029}	16,454	11,862	8,372	5,713	3,675	2,110	912	—	− 685

APPENDIX C

Fund	Date	Investments at cost as per balance-sheet (1)	Annual interest income (2)	Average yield on cost prices j (3)	Value of existing securities to yield $2\frac{3}{4}\%$ to redemption (4)	Equivalent annual interest income in perpetuity (assuming all investments after 1. 1. 47 earn $2\frac{3}{4}\%$) (5)	Value of existing securities to yield rate j in perpetuity (6)
		£000's	£000's		£000's	£000's	£000's
A	1.1.47	60,000	2400	4%	76,000	2090·0	52,250
B	1.1.47	47,000	1880	4%	60,958	1676·4	41,909
	1.1.57	62,000	2170	$3\frac{1}{2}\%$	71,465	1965·3	56,151
C	1.1.47	35,600	1335	$3\frac{3}{4}\%$	46,611	1281·8	34,181
	1.1.67	59,000	1770	3%	62,400	1716·0	57,200
D	1.1.47	24,200	787	$3\frac{1}{4}\%$	28,217	776·0	23,876
	1.1.77	60,000	1725	$2\frac{7}{8}\%$	62,001	1705·0	59,304

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ABSTRACT OF THE DISCUSSION

Mr C. E. Puckridge, in introducing the paper, pointed out that the basic theory underlying the paper applied equally well when a valuation was to be made of any company, fund or society which undertook long-term liabilities in exchange for cash payments which it invested at interest.

Mr R. I. MacIntosh, in opening the discussion, said that the main point brought out by the paper was the interrelationship between the value placed on the assets and the valuation rate of interest. He thought that perhaps the method in most general use was to fix what seemed to be a suitable value of the assets having regard to all the circumstances of the case, and then to decide upon the valuation rate of interest. That decision would be based mainly on the yield to be realized on the fund, taking into account actual interest income and the selected value of the assets, and also, of course, the trend of interest earnings. The selected value of the assets might be market value, cost price, or some other book value. He thought that in current conditions most actuaries would be doubtful of the wisdom of using market values in the valuation of a fund—and thereby recognizing possibly temporary appreciation—particularly where any securities were to be redeemed at par. The use of any value other than market value could not, however, be completely satisfactory, and probably those were the considerations which had urged the author to prepare the paper.

The method of approach in the paper was the reverse of the general method he had referred to. The author had taken as his only assumption what he considered to be the interest rate of the future, and had so adjusted the value of his assets that that assumed future rate was realized by the existing fund also. Personally, he thought that there must be general sympathy for that method if careful consideration were given to it. In a pension fund valuation extreme care was taken to place an accurate value on the liabilities, and it might be felt that the final result lost much of its meaning if the assets were valued by a more or less arbitrary method. The value to the fund of fixed-interest securities consisted of interest payments with, perhaps, repayment of capital on a fixed date. It was naturally possible to value those securities by the same method and with the same precision as in the case of the liabilities. That method, the author had claimed, produced the only correct answer, and in the light of the initial assumption that seemed to be the case. It would be apparent, however, that the individual values of different assets might vary considerably from the market values, and the author had qualified his method by the admission that it would be difficult to adopt any value in excess of the total market value. Such a position would arise if the assumption regarding future interest rates were too cautious.

He thought that the author's method became rather doubtful when shares or mortgages formed part of the assets. In the former case the choice of an annual income might be difficult, and with a mortgage at a rate of interest above the valuation rate the method would involve placing a value upon it in excess of the amount of the loan. Where the investments were confined to the gilt-edged class, however, the method became quite practicable.

It was suggested in the paper that if the proposed method were adopted less would be heard of the phrase 'only an actuarial deficiency'. He was a little doubtful, however, whether many employers would accept with complete confidence a valuation of the assets by a method which they were probably quite unable to understand. They would willingly regard the valuation of liabilities, he thought, as being entirely within the province of the actuary, and as one in which the actuary could exercise his mystic principles; but it was doubtful whether their faith would extend to the other side of the valuation balance-sheet.

He thought, however, that consideration of the paper gave more confidence to the possible use of market values, coupled, naturally, with a suitably reduced rate of interest derived therefrom. The disadvantages were that fluctuations in the values of the assets and sudden changes in the rate of interest were liable to be disturbing and to make more

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difficult the relation of the results of successive valuations. The table in paragraph 18 placed the valuation rate of interest in its right perspective and showed that whatever rate was used the true deficit in the fund could not be altered. In a stationary fund there was a fixed annual amount required to balance income and outgo, and the valuation rate of interest could not affect that in any way. It was apparent, however, that the valuation deficit might vary within wide limits and yet still allow a correct interpretation.

It seemed that consideration was called for on the part of actuaries of what a valuation deficit really purported to be. On the author's assumption of a future earned rate of interest it could be given exact definition: it was the cash sum required to be added to the fund at the valuation date to produce exact solvency. If, however, the assets were valued at cost, or, at any rate, appreciably below market values, that definition would not hold. The use of lower asset values would imply a higher valuation rate of interest, and the payment to the fund of a sum equal to the resultant deficiency would require its immediate investment at the valuation rate of interest—probably an impossible condition. The real answer was given in the paper: a valuation deficit by itself had no exact interpretation when not coupled with a valuation rate of interest. The two together, however, revealed the annual shortage of income, and the cash sum to produce solvency was the amount which invested at current rates would produce that income.

The author had done well to emphasize the expected strain which might arise from future entrants. It was normal to test the contributions payable on the valuation basis. The problem was dealt with in paragraph 6 of the paper, where $i\%$ was defined as the future investment rate ($2\frac{3}{4}\%$ in the example given). A rate of $j\%$ was currently being earned on the existing investments, and by balancing the two an intermediate rate, $k\%$, was obtained and used for the valuation.

The author had then set a rather tricky problem by asking at what rate contributions should be calculated to avoid new entrant strain. He had suggested $i\%$, the new money rate, and had then rather playfully put forward the alternative of $k\%$, the intermediate rate chosen for the valuation. Personally, he thought that that question revealed the weakness of the method. If in estimating the value of k the effect on the growth of the fund of the admission of future entrants were taken into account, it could be said that k was a low enough rate for the new contribution scale. He thought it might still be admitted, however, that the selection of that rate did not allow of such refinements, and that at best the method was very rough. If the selection of a rate of interest was the only assumption that had to be made in a valuation, some method such as that outlined by the author would become essential, but with all the other 'imponderables', as they were called in the paper, he felt that refinements based on a guess at the future earned rate should not be given too much weight.

Mr W. E. H. Hickox remarked that it might seem strange that there should still be scope for discussion on the theory of valuation, a subject which actuaries had debated for over half a century. He thought that the reason was historical. Problems of valuation had in general been discussed—and actuarial science had mainly evolved—in connexion with life assurance, where the essential criterion of a good valuation had been that it should be a safe valuation leaving margins to meet possible contingencies and to provide surplus for bonuses. Pension funds, however, were in a somewhat different position, and the retention of similar margins might bring out unnecessarily large deficiencies which would in many cases be unfair to the employers and members. The object of a pension fund valuation was usually to test the solvency of the fund, and for that purpose something more precise was required than independent valuations of assets and liabilities.

The essence of a solvency valuation was the comparison of the future income to be received year by year from contributions, interest, and maturity of investments with the future outgo to be paid year by year to meet claims and expenses. In making that comparison he thought that attention should be confined to existing business, because future entrants did not involve contractual obligations already imposed upon the fund and the rates of premium or contribution to be charged for them were as yet unknown. The problem was how to capitalize the difference between future outgo and income in

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respect of existing members. From a practical point of view the author's direct method probably gave the best solution, but from a theoretical point of view he doubted whether it was scientifically possible to reduce the problem to any one single rate of interest. Did the author's valuation rate i represent the long-term yield on new money, or the yield on that type of security in which the actuary expected the fund to be invested, or the yield on those investments which would have to be purchased if the term of the assets was to match that of the liabilities? It might be argued that the answer did not matter very much. Nevertheless, the yield obtainable on investments varied considerably with the term, and if the arguments put forward in the paper were extended to life assurance, where the duration of existing contracts was relatively short, he thought the question was important.

Again, if the actuary was of the opinion that the market rate of interest would in a few years time show an appreciable rise or fall, should his valuation rate i represent the current yield, the yield one year hence, the yield ten years hence, or the yield twenty years hence?

The difficulties arose, he suggested, because in theory the valuation rate of interest should represent concurrently all those rates at which future financial transactions would take place, and the use of one single valuation rate of interest was a somewhat artificial and arbitrary (although convenient) device. The ideal theoretical solution to the problem would be to project the future history of the fund as regarded existing members on the mortality and expense bases assumed, setting out year by year how much it was expected would be paid out of and paid into the fund, allowing for the sale of investments in those years where outgo exceeded income and for the purchase of investments in those years where income exceeded outgo. The deficiency to be made good would be that capital payment which, when invested in securities of the appropriate term, would ensure that for each future year there would be a balanced budget. Such a counsel of perfection would require the actuary to forecast the terms for the sale and purchase of investments throughout the future. Whilst that procedure was hardly practicable, he thought that it gave the theoretical solution which formed the background of the problem.

His remarks were not intended as specific criticism of the author's direct method, which was in his view a reasonable practical approach to the problem, but as a general criticism of the use of one single valuation rate of interest in all circumstances. The indirect method could be made scientific only by a roundabout method of approach which entailed a certain amount of unnecessary work. That was a serious criticism of the indirect method, but an even more important criticism was that by failing to value at the market rate of interest the picture was confused and the truth obscured. In that connexion he wished to mention a point which, whilst it concerned pension funds, was perhaps of greater importance to life offices.

The trend of recent investment conditions had been towards a reduction in the current market rate and a diminution of the margin between the net earned rate and the valuation rate. At the same time there had been an increase in market values and an augmentation of the capital margin between market values and book values. Though that margin could of course be applied to meet the cost of reducing the valuation rate of interest to the current market rate (less tax), the uncertainty of future rates of interest and possibility of fluctuations in Stock Exchange prices provided sound reasons for retaining at least part of the capital margins and either postponing the change of valuation basis or making only a partial change. It should, however, be remembered that the market values of redeemable securities standing at a premium, such as the majority of British Government securities, would gradually approach par and the excess over book value would run off year by year until at redemption it would have disappeared entirely. If, then, the average term of the investments was less than that of the liabilities, the capital margin might disappear too quickly and too much interest surplus be allowed to emerge in the early years.

If existing business only had to be considered the risk might not be great; but, if new business was also valued at the relatively high rate of interest used for existing business, surplus would be over-released unless each year sums were set aside sufficient to meet

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the deferred cost of change or fresh capital margins were created. If, for instance, an office was in a stationary condition and the whole of its capital margins was needed to change its valuation rate of interest, it would be apparent that failure either to reduce the valuation rate or to counteract the disappearance of the capital margins on redeemable securities might mean that it would not be possible to strengthen the valuation basis sufficiently at a later date.

Mr C. A. Poyser observed that so far the discussion had revolved round the value of the assets; he, however, wished to refer to the other side of the picture. On a first reading the paper might appear to be a theoretical exercise in equivalent values and emerging costs, but it was as a contribution to a practical problem that the paper, as he understood it, was put forward. On the general thesis of the paper, he agreed that the assets of the fund should be valued on a basis which corresponded to that adopted in valuing the liabilities, although there might be scope for argument as to the precise way in which that should be done. In a pension fund, unlike a life assurance fund, there was no need to retain substantial hidden reserves, and what was proposed, in effect, was that credit should be taken for the interest to be earned in the future in excess of the valuation rate on the existing portfolio of investments. He knew of a number of cases in which such an adjustment had been made; there was particular need for it under prevailing conditions, since often the running yield on the fund was substantially above the rate which the fund could earn on new investments. If the actuary decided to fix his valuation rate somewhat above the current market long-term rate, in the hope that the trend of interest rates would turn, it would be necessary, presumably, to allow for some deficiency of interest on future investments as well as to take credit for the excess interest on the existing portfolio.

In the particular fund considered in the paper, the members paid 5% of salary and the employers paid the balance of the contribution required, so that it did not make any difference to the total liability to be met by the employers (*a*) if the contribution for new entrants remained at the old rate and the whole strain in respect of new entrants was shown as a deficiency to be met by the employers, or (*b*) if the employers' contribution in respect of new entrants was increased to such a figure that no strain emerged. The total liability of the employers was the present capital value of their future contributions expressed as a percentage of salary together with the payment required in respect of the deficiency, and the precise manner in which the deficiency was met did not affect that total. The deficiency might be met by payment of a perpetuity—the interest on the deficiency from year to year—or by an additional contribution expressed as a percentage of salary.

The position discussed was, however, of a somewhat theoretical nature; it was more usual for the contributions to be shared equally or in some fixed ratio between the employee and employers, and for the trust deed of the fund to specify the method by which any deficiency should be made good, e.g. by adjustment of the benefits or of the contributions of both employers and employees. In such a case, the total liability of the employers was no longer a fixed amount, but would depend to some extent on whether the rate of contribution for new entrants was changed. If the rate of contribution was increased to that which the actuary considered adequate on the basis of the long-term rate of interest, part of the strain from new entrants was passed over to the members. It would seem most undesirable that the prospective deficiency from future entrants should be shown as a deficiency in the valuation balance-sheet, when merely by a change of policy on the part of the employers—if indeed they had any option in the matter and the trust deed did not bind them—they could remove that deficiency by requiring the payment of an adequate rate of contribution. Personally, he thought that the actuary should insist that what he considered to be the correct rate of contribution should be paid by future entrants, just as a life office would not indefinitely enter into new contracts at rates of premium which would involve a loss to the office.

In the case of local government funds, where the benefits and contributions were laid down by statute, the rates of contribution were probably inadequate under the prevailing financial conditions and the deficiency had to be made good by the administering

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authority. Whether the liability was made good by means of a series of equal annual charges, set up as the deficiency emerged at successive valuations, or of a perpetuity, calculated on the capitalized present value of the expected deficiency in respect of all future entrants on some assumed basis, did not seem to him to be a matter of great moment; it was merely a rearrangement of the financial liabilities.

Mr R. W. A. Fowler said that in making a valuation of a pension fund the first duty of the actuary was to determine bases for his calculation which in his judgment were most likely to reflect future events, and to make his assumptions in a precise manner. That having been done, there was in his opinion only one correct answer to the question what annual sum or single equivalent was required such that the fund was and would continue to be exactly solvent provided that the assumptions proved to be correct. In the paper the assumption was made that future investments would yield $2\frac{3}{4}\%$, and the implications of that assumption were carefully analysed.

There appeared, however, to be a tendency on the author's part to assume that it was easier to guess the average rate at which new money could be invested, not merely at the valuation date but for the whole future of the fund, than to guess the average rate to be earned by the fund as a whole over its future existence. The author had even gone so far as to suggest in paragraph 26 that there would be partial justification for neglecting the deficit of some £7,000,000 under the $3\frac{1}{4}\%$ valuation. That would be so if the guess of $2\frac{3}{4}\%$ for future investment proved to be right. On the other hand, the actuary who assumed $3\frac{1}{4}\%$ to be the average rate over the whole future life of the fund might equally well prove to be the more correct in the event. The fact was that the two assumptions were quite different, and nobody could say which of them was likely to be the more correct. To his mind, the '$3\frac{1}{4}\%$ whole fund' assumption was just as precise as the '$2\frac{3}{4}\%$ new money' assumption so far as definition was concerned.

If it were possible to forecast with a reasonable degree of accuracy and facility the average rate of interest at which all future money could be invested, it would obviously be better to approach valuation from that angle. In fact, however, such a forecast could only be a guess so far as any period but the immediate future was concerned—a guess which was as difficult as, if not more difficult than, forecasting the continuing yield on an invested fund. In those circumstances, he could not help feeling that there was still likely to be a preference for the more usual form of assumption of an average yield on the whole fund based on the value of the assets taken for the valuation, particularly since many funds would have in their portfolio investments of the types referred to in paragraph 34 which could not be dealt with in a precise manner.

So far as the valuation of assets was concerned, he thought that nobody could quarrel with the basis of market value subject to a maximum of the redemption value. The basis of cost price, appreciation being excluded, gave a rather more conservative approach, which many would favour. In any case, having decided on the basis and obtained the running yield, the actuary would then obtain his valuation rate by an educated guess which, even if he did not consider its implication in regard to future investment yield, seemed just as likely to be correct as that obtained by the author's approach from the new investment angle.

Mr G. Heywood remarked that one of the assumptions which was implicit throughout the paper was that, having determined the rate of interest which a fund might be expected to earn in the future, it was possible to arrive at what the author had called the correct answer to the valuation. That answer was produced by a method which was rigidly defined, and it was suggested that if the actuary wished to depart from that method—for instance, by taking assets at book values or by making no specific reserve for strain through the admission of future entrants—he should still produce the same result by using an arbitrary rate of interest appropriately determined. Personally, he felt that it was not possible to be so dogmatic, nor was it possible to lay down a procedure which was the same for every fund and which produced what was called a true result. The theory underlying the author's method was perfectly sound, and might well be generally applicable for stationary funds in an actuarial paradise where the future

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experience of every element entering into a valuation, including the number of future new entrants, never deviated from that expected. In practice, however, conditions were very different and the word 'margin' immediately made its appearance. Therefore, even when i, as defined in the paper, had been fixed, the rate to be used in the valuation, even in what was called the straightforward method, could not be determined without a consideration of every other element entering into the valuation basis; it was also necessary to take into account the type of the fund and the purpose of the valuation.

For example, in the case of a fund where the numbers were small, it might be advisable to retain comparatively large margins in the valuation basis, including the valuation rate of interest, to offset the possibility of large fluctuations in future experience owing to smallness of numbers. On the other hand, in the case of the fund of a local authority, where the benefits payable from the fund were in effect guaranteed, since all necessary costs were charged on the rates, the actuary might be satisfied with a smaller margin in the rate of interest. In fact, for a fund where the benefits and contributions were fixed by statute, whatever the actuary might assume could not alter the ultimate cost, which for a given rate of interest was an arithmetical fact. All that he could do by varying the valuation basis was to say when and how the money would have to be paid to meet that cost.

Regarding the valuation of investments, he personally favoured whenever possible the traditional basis of book values, but there were many cases where some relief might be given by valuing assets on the basis indicated in the paper and writing them up accordingly. No method, however, could be of general application. For example, in the case of a valuation for the purposes of winding-up, the only value to be placed on assets would seem to be the market value. On the other hand, in the valuation of a continuing fund, if instead of taking book values it was decided to value the assets at a rate of interest, then that rate of interest might not necessarily be i as defined, but would depend upon every item which entered into the constitution of the portfolio.

As implied by the author, British Government securities and ordinary shares involving some risk could not both be treated on the same basis; there was also the problem of money deposited on mortgage and repayable at six months' notice, where the absence of a redemption date might make the determination of the equivalent annual income in perpetuity a difficult problem.

The author's method as a whole recalled the original form of the bonus reserve method of valuation of the liabilities of a life assurance company, in that it endeavoured to estimate every element entering into the valuation basis as closely as possible. In the case of pension funds, he felt that the method was a product of the times, with their low interest rates, high salary scales and the ever-decreasing mortality of pensioners, and had arisen as an effort to give every possible relief to the deficiencies which were emerging in most pension fund valuations.

Finally, therefore, it was of interest to consider whether the method would still apply in a period of higher interest rates. A few years ago, i might well have been estimated at $3\frac{1}{2}\%$, and the average yield on a fund might have been over 4%. To have valued at $3\frac{1}{2}\%$ and written up the assets would probably have yielded a surplus which might have been disposed of by an increase in benefits. Would it not have been more prudent, in the light of recent events, not to have produced that 'correct' answer, but to have valued at, say, $3\frac{1}{4}\%$ and taken the assets at book values, and thus to have retained a most useful margin against the subsequent less favourable conditions?

Mr R. J. W. Crabbe commented that the opener seemed to have taken it for granted that there would be fairly general acceptance of the proposition that a correct valuation could be arrived at by the method of equating the assets and the liabilities at the same rate of interest. The trend of the discussion had shown, however, that the proposition could not be taken for granted unless there were some justification for it. His own feeling was that to talk of estimating a rate of interest which could be earned permanently on new investments and of the use of such a rate as the justification for the suggested method of valuation somewhat obscured the issue. Mr Hickox had pointed out that for a pension fund, as opposed to a life office, the valuation was really a solvency valuation; in other words, there was no object in retaining large margins and con-

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sequently confronting the employer with a substantial deficiency when what should really be aimed at was to charge the employer with the correct amounts, as nearly as could be estimated, as the fund progressed so as to enable it to maintain a permanently solvent position—not a position in which large surpluses would emerge at later dates.

If that were granted, then the position was that a rate of interest had to be determined which would make it possible to equate foreseeable liabilities and foreseeable receipts; and of those foreseeable receipts the excess interest on higher-yielding securities was as concrete as any other receipt. The problem became that of deciding what rate of interest should be used to equate them. One suggestion was that the actuary should guess or estimate the average rate which the fund would earn over the future period; but what future period? Presumably over the future lifetime of existing members. That, of course, was treating the fund as closed. If that rate turned out to be higher than the rate actually earned by new investments, there would definitely be a deficiency for every new entrant, whether or not it was decided to take account of that deficiency.

The author's solution was to take the current market rate as the rate at which to equate assets and liabilities. That had certain definite advantages; for instance, it was helpful in deciding what value to place on the assets. Personally, he suggested that an appropriate way of valuing the assets was an amortization basis, subject to appropriate adjustments where the security for future interest, etc., was less adequate than in the case, say, of British Government securities. That was, in fact, one way of describing the current market price, for the Stock Exchange value of a security was in effect arrived at by such a process. By adopting the current market rate of interest for the purpose of the valuation of both sides, therefore, the market price provided a fairly adequate measure—he knew that there were shortcomings—of the value of the assets, upon the same basis as was used for the value of the liabilities.

On that basis also the actuary was in a position to say what annual cost would be involved by new entrants. His figure might be wrong, but it was no more likely to be wrong on that basis than on any other basis which could be adopted, and at any rate it had the virtue of consistency with his valuation. If he were to adopt a rate of interest in excess of the current market rate, he would have to make a difficult decision with regard to new entrants; he would have to say 'I know that if present conditions continue—and they are obviously likely to do so for a time, if not indefinitely—I am certain to have a strain in respect of new entrants. What am I going to do about it?' He thought that most actuaries would say that the correct thing to do was to make new entrants self-supporting in the near future, as far as could be foreseen; that involved the use of the current market rate, or something near it.

On those grounds, it seemed to him that what had been suggested by the author was not a refinement on the guess of the future earned rate of interest, but an attempt to give a closer degree of certainty in equating assets and liabilities than could be obtained by the ordinary method. Whether or not it was decided to use that method in the valuation report, its value still remained in that it gave an instrument whereby it was possible to decide what were the basic assumptions underlying the particular rate of interest used. If, for example, it was decided to use an intermediate rate of interest between the rate earned and the current market rate, the analysis given in the paper showed that so long as j, the current yield on the fund, exceeded the valuation rate, k, there would be a yearly profit; but it also showed that that profit was not available for distribution, because if the current market rate did not rise, but still remained at i, eventually the fund would yield i, and the valuation rate would have to be reduced to that level. Clearly, therefore, any surplus interest earned while the valuation rate was k would have to be used as a reserve to enable the rate to be reduced to i. The paper gave a method first by which k could be fixed, and secondly by which the actuary could decide at what rate of progress the valuation rate of interest should be reduced so as to reach the rate of interest i by the time the current yield on the fund had fallen to that rate.

Mr E. J. Lancashire referred to the position of friendly societies, and said he felt that for such societies circumstances might well arise in which action on the lines suggested in the paper would be necessary although for very different reasons. It was

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very rare indeed to find a friendly society where an employer had given any sort of guarantee, and it was comparatively rare to find an employer interested financially in a friendly society. He had recently, however, seen the valuation of a friendly society which consisted of two sections: an old closed section with funds of about £1,000,000, and a new section with funds of about £250,000. The valuation rate of interest had been reduced in recent years, but it was decided to make a further reduction to 3 % on the occasion in question, because a large proportion of the funds was due for redemption in the ensuing ten years, though the current yield on the investments as a whole—there was no separation of assets—was £3. 17*s*. 6*d*. %. There was a rather peculiar provision that any surplus disclosed had to be transferred to a surplus reserve fund, so that no surplus was brought forward from the preceding quinquennial valuation and the strain in passing to the lower valuation rate of interest had to be met out of current profits. That was possible in the old closed section but not in the new section. By valuing at 3 %, the new fund would have been in deficiency, and the question arose whether it was right and proper to show the new fund in deficiency when there was an asset consisting of the excess interest which it was clear would be earned over the next few years even if there were to be a further fall in the market rate of interest.

Taking into account the high current yield, the future prospects, and the undoubted stigma attaching to a deficiency in the minds of the members of a friendly society, it was decided to take credit in the valuation for a portion of the excess interest to be earned over the ensuing valuation period, and as a result the new fund was shown as possessing a small surplus.

He felt that if action were taken on the lines he had described, and especially if credit were taken for the excess interest over a very long future term, it was important to warn the members of the society—he was speaking of friendly societies rather than pension funds—that credit had been taken for a future asset, and that as a result the surplus shown in the future would be affected. Friendly societies did not often advertise their bonuses and they did not canvass on bonus prospects; nevertheless, they had what in effect were bonus schemes, and many of them relied on future surplus for the fulfilment of those bonus schemes. It was important to give a warning, therefore, that the flow of surplus had been at any rate interrupted.

He felt, like Mr Hickox, that the most satisfactory way of dealing with the problem would be to vary the rate of interest assumed to be earned in each future year, taking account of the way in which the fund would increase and the rates at which the existing investments would fall to be reinvested. Whether it was possible to arrive at such a method of valuation he did not know, but it would have the advantage that the actuary would be left free to deal with the value to be placed on the assets without so much regard to the question of excess interest. In dealing with the matter in the way suggested by the author, there was some element of risk that the actuary with his eye on the income might overlook what might be called the intrinsic worth of the investments. He hoped that it might be possible to devise such a system of valuation and, if so, he thought that it might be called the 'budget-years-passed-through' method.

Mr S. J. Rowland, in closing the discussion, said that he was a little uncertain of the extent to which the author had meant members to discuss the paper. At the end of the first paragraph the author had thrown out a wide invitation by saying: 'It is in the hope of provoking an exchange of views on the best method of dealing with this present situation that this paper has been written.' At the end of the paper, however, the author had narrowed the issue down to the relatively small one of how, having decided on the rate of interest on future investments, the particular valuation should be carried out. He proposed to accept the earlier invitation.

The question raised was of the utmost importance at a time when interest rates were so low. It should be borne in mind that pension funds provided for the old age of people who had been looking upon the income which they hoped to receive as guaranteed: it was the duty of actuaries to give them accurate advice and not to fail them. In the case of an ordinary life fund, there was the cushion of the with-profit policyholders and also, in many cases, the shareholders' capital; but with a pension fund there was no such cushion.

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With regard to the existing contributors to the fund, he thought that, either directly or by implication, the solvency of the fund was usually guaranteed by the employer. For new entrants obviously an entirely new contract could be entered into. The author had discussed the $2\frac{3}{4}\%$ rate, but so far as contributions from new members were concerned it must be borne in mind that the contribution from the employee was 5% of his salary, so that the whole cost of the fall in the rate of interest as applied to new members would fall on the employer. That increase was a very material one. In Fund A it amounted to an increase in the contribution of 45%. It had been suggested that the first thing that the actuary would do on starting a valuation of such a fund would be to consider his rates of interest and so forth. He personally would not do that at all. He would assume in the circumstances under consideration that, whatever reasonable valuation basis was used, the fund would show a deficit, and, as dealing with the deficit was largely a matter of policy to be decided by the employer or by the employer and employees together, he would in the first place forecast such a deficit and then suggest the various means of dealing with it; because on the method that might be found most suitable for dealing with the deficit depended the most suitable method of valuation.

The discussion had been very much bound up with the question of the current fall in interest rates. In the very late 1890's the rates of interest on gilt-edged securities had been just as low, or very nearly so. They had varied tremendously since then; they might equally vary, in spite of Treasury control, in the future. That control, for instance, might not be maintained. It seemed, therefore, that whatever method was adopted should be flexible. The surplus which might be brought out, if in fact the actuary used a low rate of interest and rates of interest were to increase in the future, would not in the usual case accrue to the employer, in spite of his increased contributions.

The best solution, at any rate with regard to existing members, would be for the employer to guarantee the rate of interest on which the contributions were based. The troubles of the actuary would then be over, because he could value at that rate of interest. The deficit in interest was the actual measure of the deficit in the fund, and therefore the guarantee of the rate of interest was a method which took care of the future deficit of the fund completely. It had as a rule the disadvantage that it would mean an increasing payment by the employer, which was probably an undesirable feature; but it could be overcome in many ways, for example, by 'making up' on the purchase of stock which did not yield the appropriate rate of interest, the employer providing an additional amount of stock sufficient to bring the interest up to the requisite figure—or the employer could guarantee the rate of interest in respect of existing contributors only.

So far as new members were concerned the position was entirely different, for it was possible to enter into a new contract altogether. It would certainly be wise to reduce the rate of interest used in the calculation of contributions for new members, but probably not to the low level of current interest rates, in view of the possibility of upward variations in the future. If, for the sake of argument, contributions were calculated on a $3\frac{1}{4}\%$ basis, and the employer were to guarantee that rate of interest, the position was the same as before, if the employee only paid 5% of salary and the balance fell on the employer. On the other hand, however, there was no reason whatever why the contribution of the employee should not be increased from 5% to 6% or 7%. That would remove the deficit and the uncertainty of valuation, and the cost could be equitably arranged between employer and employee.

As the author had said, the true deficit was not altered by the method of valuation. The valuation was merely an approximate stock-taking at a particular time with the best estimates which could be made of future rates of mortality, interest and so forth. As had been suggested in the discussion, if actuaries were omniscient they could obtain the true deficit which would in fact be experienced in the future. Being mortals, they could not do that, but a guarantee by the employer removed one difficulty entirely. It might be felt that he had tended to elaborate that point, but it seemed to him to be a means which might be available in a large number of cases of getting over the main difficulty.

The author's method was, in effect, a bonus reserve method of valuation, and that had a very respectable pedigree in the profession, particularly if it was considered that in effect column (4) of Appendix C could be interpreted to be the present market value of

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the securities in the existing fund, in which case the table in paragraph 18 would reproduce exactly the same adjusted fund at the end. Item (iv) would be increased to £76,000, the reserve required to maintain the rate of interest would be modified, and the resulting adjusted fund would agree with item (vii) in the table.

It should, of course, be remembered that the amount to be invested each year would not in practice be as large as might appear at first sight. The income from the contributions was first used to pay the pensions and other current outgo, and it was only the balance of income over expenditure which needed to be invested; that, in many cases, would be a relatively small proportion in each year.

A problem which might create considerable difficulty would arise in the event of the closing of the fund, namely the effect of the high premiums on the investments when they had to be realized to pay the pensioners as the contributions ceased and the fund diminished from year to year.

In conclusion, he agreed with the author's suggestion of taking the market value of the securities in place of their values on a 2¾% basis. Such a procedure was reasonable on the ground that the investments were realizable at the time of the valuation at their market value and could be reinvested in marketable securities to yield the rate used in the valuation. He also agreed that in setting out the results of a valuation on such a basis, carried out with a view to ascertaining a definite deficit which could not be met by guaranteeing the rate of interest, the author's method was most suitable.

The President (Mr A. H. Rowell), in proposing a vote of thanks to the author, said that he had set a good example to other writers of papers by reminding readers, towards the end of the paper, of the precise nature of the question for discussion.

He thought that the author had placed himself on very firm ground when he advocated a method of valuation designed to ascertain the real and not any theoretical deficit—real in the sense that the actuary would presumably be prepared to see the fund receive that same sum of money in order to restore its solvency, and would do so without too many qualms. Mr Puckridge had earned the gratitude of his colleagues by digging down to the roots of one aspect of pension funds which had not received sufficient attention in the past.

Mr C. E. Puckridge, in reply, said that there was one matter that had emerged during the discussion which had taken him by surprise. It was apparent from his paper that he had taken it for granted that an actuary called on to report on the financial position of a pension fund would of necessity make up his mind to assume that a certain rate of interest was likely to be earned on future investments. It was now clear to him that he was wrong in having done so; but, nevertheless, he maintained that in his paper he had demonstrated that the assumption of an average rate of interest for existing and future investments, taking existing investments at some fixed value, might easily produce a result which could not possibly be arrived at by using any reasonable assumption for the rate of interest to be earned on future investments. He saw no reason why any particular significance should be placed on the value of the investments appearing in the balance-sheet. That might be cost price, or some other figure that might happen to have been shown by the accountants; he said that advisedly, because some accountants brought in profit on realization of investments while others carried the item to an investment reserve account or used it to write down other investments. If a fund had certain investments, it did not matter at what price they had been purchased; it was the value of the securities according to the valuation basis that mattered.

Turning to p. 10, and considering valuation (*b*), where it had been assumed that contributions would be adjusted so that future entrants became self-supporting, the actuary who adopted the indirect method (as he had called it) and who used a rate of £3. 17*s*.% was definitely assuming that it would be possible to earn 2¾% on future investments. There were certain assets which, if held to redemption, would produce a known income in the intervening years and known sums of money at the redemption dates; the only matter in doubt was the rate of interest obtainable on future investments

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when money became available either as a result of redemptions or because contributions and interest exceeded outgo for benefits. He thought it would be accepted that, when the actuary decided to say 'I will take credit for existing investments at a certain figure and I will assume that on the average a certain rate of interest will be earned in the future', he had, whether he admitted it or not, made a definite assumption regarding the rate of interest that would be earned on future investments.

Attention had been drawn in the discussion to the fact that in the examples in the paper the employee paid 5 % of salary and the employers paid the balance of the contribution. He was sorry that he had introduced that feature, the only reason for which was to simplify the calculation of the liability in respect of future entrants. The theory remained unchanged if contributions were divided equally between employers and employee and if the method of dealing with a deficiency was laid down in the rules of the pension fund.

The suggestion had been made by the opener that an employer might not accept with confidence an actuary's valuation of assets, while regarding the valuation of liabilities as something rather mystic with which the actuary could deal. Personally, he thought that if an employer was presented with an actuary's report in which the liabilities had been valued in some mystic manner which he did not understand, and in which credit had been taken for existing investments at a book value which he knew to be materially less than the market value, he would say to himself: 'There is a nice little margin here; I do not think that I need do anything about this, or at any rate I can bring in this margin for which no credit has been taken.' The employer would not easily be made to understand that the actuary, by his mystic processes, had already taken that margin into account. He himself had in practice adopted the method of revaluing existing investments and had explained to employers what he had done; they had seen without difficulty that the margins had already been used and that the deficiency was real and should be dealt with. He knew of at least two cases where an indirect method of valuation had shown a large deficiency and no action had been taken. The accountant had told the employer: 'You need not worry about this deficiency; we have in hand the difference between the market value of the assets and the value for which credit has been taken by the actuary.'

Mr Poyser had mentioned that it was possible to deal with the difference between the book value and the true value of investments by bringing in the value of future interest in excess of the valuation rate. If that adjustment was to be complete the profit or loss on redemption should also be taken into account.

Mr Rowland had referred to guaranteed rates of interest. Personally, he did not like the idea of asking employers to guarantee rates of interest. At one time rates of interest commonly stood at about 4 %; employers had been asked to guarantee that rate, and in some cases had done so. The result, in the case of a rapidly expanding fund for which such a guarantee had been given, was that an unreasonable burden was passed on to posterity. Although, perhaps, the employer had so far had to pay very little excess interest, it was possible that in twenty or thirty years' time very large sums of money might be required in respect of the guarantee.

Mr W. F. Marples has sent the following written contribution:

An actuarial report to the managers of a pension fund should be based on ground common to both parties. Much of the misunderstanding that has been known to arise would have been prevented had care been taken in the early stages to establish this ground of common understanding. In most cases the understanding is arrived at in discussing the policy of investment and valuation of the assets of the fund. From this base, an attempt is made to build up a picture of the effects of the experience of the fund, so that in due course the managers accumulate knowledge of the working of the fund which enables them to make an intelligent guess regarding the effect of any abnormal experience upon its finances. On this point I would deprecate the use of contribution rates taken to 3 decimal places and interest rates calculated to 4 decimal places. One cannot in the working of a fund justify the degree of precision which is indicated by such figures.

The author employs the equation:

$$\text{Contributions} + \text{interest} = \text{Pensions} + \text{other outgo}$$

(which theoretically is true when the fund arrives at a stationary condition) in producing his ultimate annual interest deficiency, and demonstrates that it is independent of the valuation rate of interest. Does not this suggest that the cheapest way of meeting a deficiency is to guarantee the rate of interest? It may be noted that the stationary condition is hardly ever met among pension funds, since all the other variables such as salaries, withdrawals, pensioners' mortality and the rate of interest may upset the position even if the numbers indicate that it is precariously balanced at the stationary point. I am not in favour of a guarantee of the rate of interest for another reason which I mention below, but I think it has to be faced that the guarantee may be the cheapest method of financing the fund since interest is provided in perpetuity without any accompanying sinking fund. The guarantee of contribution rates might follow from the guarantee of interest rates; but, in fact, it need only apply to the rates of contribution paid by the current active members. From the point of view of the employer, the real cost of employing a staff is the total sum to which salaries, wages and pension charges amount, and if this aggregate sum is kept in mind an increase in contributions to the pension fund falls into perspective in a way which it does not if only the payments to the pension fund are considered.

The author, unless I mistake him, points out that the ultimate annual interest deficiency is constant and proceeds to capitalize it at various rates of interest. But why do that? I prefer whenever possible to maintain my argument on the level of annual costs and not of capital values. Capital values are a technical by-product of the organization of the pension fund, and it is necessary to consider them at various stages. But the figures which are real to the employer are the annual costs, which will include his normal contribution to the fund and any annual payment by way of guarantee or liquidation of capital values. I well remember being told of a large hospital whose management committee was appalled at the capital cost of instituting a pension fund and, in fact, dropped the idea. The annual payment to provide the initial capital over a period of forty years could have been expressed to the committee as a comparatively small sum per bed per week for that period. The amount was, I think, under 1*s*.

I do not feel that the author's suggestion of presenting a report to the managers of the fund in such a form that the deficiency on one basis is bolstered by an argument on an entirely different basis really carries conviction. I have a great respect for the real capacity of the managers of pension funds. I think they would very quickly feel their way to what I hope I may term without offence the inherent insincerity of the presentation. With an acute mind asking a few awkward questions it might be very difficult to sustain a clear case. I would far rather develop my argument on the rate of interest in the normal way indicated by D. A. Porteous's comments in his book, *Pension and Widows' and Orphans' Funds*. The managers of a pension fund will have directed a great deal of their attention towards their investment policy and this is one aspect of their duties which they will feel they understand. They are accustomed in financial matters to value the assets in a balance-sheet at cost, or book values if lower, and if a break is made away from this standard I am sure that a feeling of insecurity will be engendered which may have the extremely dangerous result of shaking their confidence in the whole actuarial report. Moreover, the aggregate of cost prices represents the total of the money accumulated by the fund and should be sufficient to meet its liabilities. It is the higher estimate of liabilities on account of variation of experience which now produces the demand for bringing some part of any appreciation to the relief of the employer. If the full plan of valuation suggested by the author is applied to all the assets of a pension fund, the result may be to produce wholly unnatural estimates of the values of the individual items and may even produce values much greater than the market values. I note that the author proposes to take the capital issues of industrial companies, including debentures which may be without date of repayment and preference or ordinary stocks, on a basis not differing by much from market values.

I admit the plausibility of his case if the assets are wholly in trustee securities with

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appropriate dates of redemption, but I do not see what advantages it offers in connexion with all types of security without redemption dates. Moreover, I would still hesitate in any report to value the fund at the equivalent of 'market prices or higher'. This brings me immediately to the 'winding-up' aspect of a valuation. If the securities representing the assets have been valued on the basis that the pension fund is to be regarded as a continuing entity, then there may be some justification for a partial use of the author's method. If however, it is to be recognized, as I feel it should be, that there is a risk that the organization may cease to exist, it is not safe to value the assets in the balance-sheet at a figure higher than their market value. In fact a reasonable margin should be deducted from this figure in order to allow for possible fluctuations in value within the valuation period and for expenses of realization. The 'winding-up' aspect is not such a theoretical consideration as might be considered; I have recently had to advise on the winding-up of three pension funds and one friendly society, and there may be more of these to follow if the reorganization of the commercial life of this country continues to be pursued in the way it is being pursued at present. I would add that the 'winding-up' aspect is my serious objection to the guarantee of a rate of interest.

I also disagree with the introduction into the valuation balance-sheet of the capital value of the strain due to the admission of new entrants at inadequate rates of contribution. Where the valuation has been made on a basis which involves a change in the rates of contribution, it is better to comment on this aspect at the end of the report. Pension funds are already hard hit by increases in the salary level and by a reduction in the rate of interest, and if the deficiency be further piled up to the rather staggering heights indicated by the author, the result would be not only to shake the faith of the managers in the reorganization of their scheme but seriously to hamper them in their future management of the fund. There would in fact be a crippling loss of confidence as well as a crippling deficiency. Moreover, is such a move really necessary? Valuations are made at the end of each five years, and if the matter is put fairly and squarely to the management that the continuance of the existing rates of contribution will cause a deficiency at the next valuation they may be prepared to deal with the matter on that basis for the sake of retaining the current rates of contribution. If they are not, then the alteration of the rates of contribution to the new members is not a matter of great difficulty. I feel that to place in the balance-sheet the capital value of the strain of new entrants in perpetuity would, when the full explanation was appreciated by the management, seriously strain their acceptance of actuarial theory. Would not the just conclusion of the managers be that the only course of action to save the fund is to cease to admit new entrants? The logical development of such a suggestion would be to start a series of funds and close them at regular intervals which would be absurd.

I feel that the whole of the author's arguments, ingenious as they are, depend upon the suggestion that the future rate of interest is very considerably lower than the rate which is being earned on the current assets of the fund. Is this situation bound to continue? I feel that if the rate of interest were to rise to, say, $3\frac{1}{2}\%$ much of the work which the author has put into his paper would no longer be applicable.

To conclude, I address myself strictly to the question propounded by the author at the end of his paper. My answer is that:

(1) I prefer to develop the reasons for my decision on the normal lines indicated by Porteous;
(2) I do not agree that the 'true' position is necessarily indicated by a valuation at rate i;
(3) I would not feel able to adopt a method of presentation involving a valuation rate of interest so chosen that the result at rate i is produced when new entrant strain is included in the statement of liabilities.

Mr Puckridge has subsequently written as follows:

The discussion revealed that some members were not prepared to concede that there is any necessity for an actuary who undertakes the valuation of a pension fund to assume a rate of interest to be earned on future investments. These additional notes have been

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prepared in an attempt to demonstrate that a decision in regard to the rate of interest to be earned on future investments is not really avoided by taking credit for existing investments at some fixed value and employing an average rate of interest to be earned on existing and future investments.

The actual investments of Fund A, details of which were not given in the paper, were:

£35,507,000 4% Stock redeemable at par in 1960,
£35,625,000 2¾% Stock redeemable at par in 1950.

the book value of which was £60,000,000.

Valuing these securities to earn the valuation rate of interest to redemption, the following results are obtained:

Valuation rate of interest	Net liability for existing members and pensioners (1)	Value of existing investments (2)	Surplus (2)–(1) (3)
%	£000's	£000's	£000's
2	87,917	80,050	−7,867
2¼	83,247	78,664	−4,583
2½	78,820	77,312	−1,508
2¾	74,704	76,000	1,296
3	70,906	74,711	3,805
3¼	67,421	73,456	6,035
3½	64,191	72,231	8,040

It is considered most unlikely that the average gross rate of interest to be earned on future investments would be taken at the present time at a figure higher than 3% or lower than 2½% and it is thought that no actuary would knowingly produce a valuation deficiency or surplus which resulted from the assumption that the rate to be earned on future investments would be less than 2% or greater than 3½%. If this be conceded it follows that the valuation bases must be suspect if a valuation of Fund A shows a deficiency greater than £1,508,000 or a surplus greater than £3,805,000. If a deficiency greater than £7,867,000 or a surplus greater than £8,040,000 should result the answer must be regarded as unreasonable.

The graph appended (p. 30) indicates the effect of the valuation rate of interest on the surplus of Fund A when credit is taken for existing investments at

(*a*) their value to earn the valuation rate of interest to redemption,
(*b*) £60,000,000, which is the book value shown in the paper,
(*c*) £65,000,000,
(*d*) £70,000,000.

A study of this graph will make it clear that, when credit is taken for investments at £60,000,000, a result which must be regarded as suspect is produced when a valuation rate of interest less than 3·72% or greater than 4·20% is used; the adoption of a rate less than 3·22% or greater than 4·63% will lead to a result which is considered unreasonable. Similarly, when credit is taken for existing investments at £65,000,000 or £70,000,000, suspect results are produced if rates of interest less than 3·32% or 2·96% respectively, or greater than 3·75% or 3·34% respectively, are used; and rates of interest less than 2·87% or 2·56% respectively, or greater than 4·14% or 3·69% respectively, lead to results which are considered unreasonable.

An actuary valuing Fund A would arrive at a deficiency of about £4,500,000 by any of the following methods:

(*a*) valuing all assets, including investments, and liabilities at 2¼%,
(*b*) taking credit for investments at a value of £60,000,000 and valuing all other assets and liabilities at 3½%,

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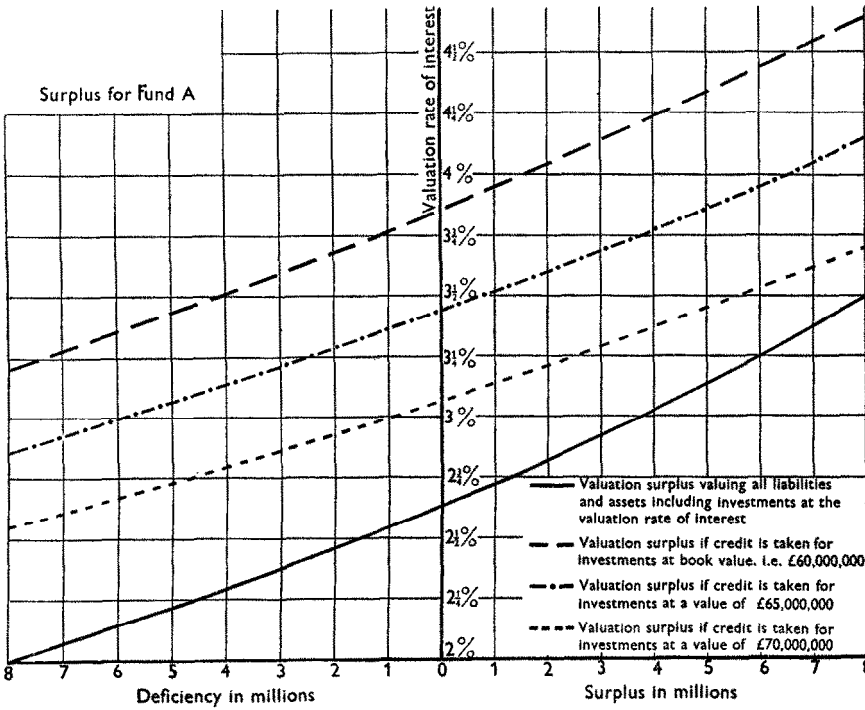
(*c*) taking credit for investments at a value of £65,000,000 and valuing all other assets and liabilities at $3\frac{1}{8}\%$,

(*d*) taking credit for investments at a value of £70,000,000 and valuing all other assets and liabilities at $2\frac{3}{4}\%$.

If method (*b*), (*c*) or (*d*) be used it remains a fact that the future rate of interest required to be earned on reinvestment is $2\frac{1}{4}\%$.



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